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1826

1955

American Agriculture—A Great Past, A Greater Future
EZRA TAFT BENSON

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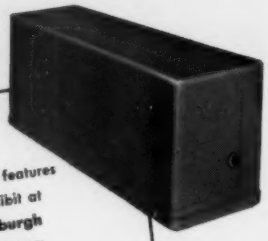

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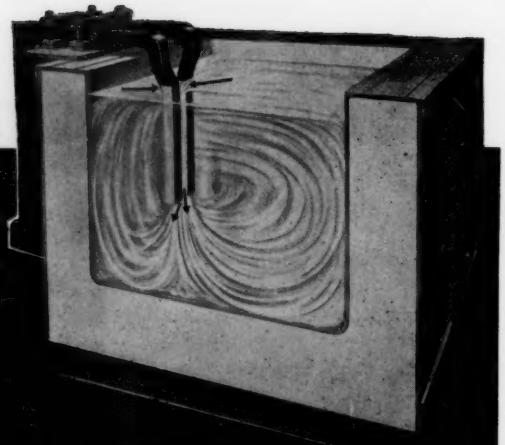

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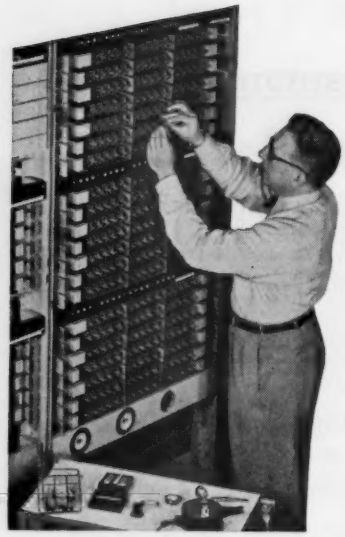

Above, Bell Laboratories microchemist applies plastic disc in heated clamp to relay contact. Imprint reveals contours of surface and picks up contaminants, if any. Part of portable test set is shown on table. Contacts, shown in small sketches, are of precious metal fused to base metal.

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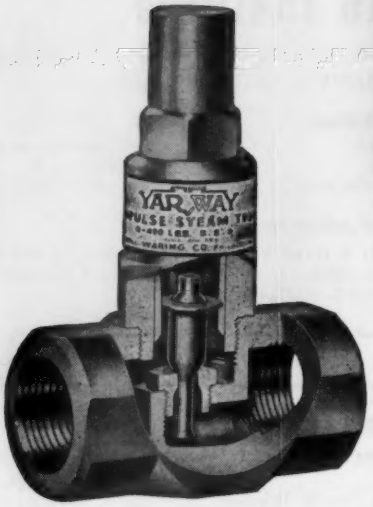

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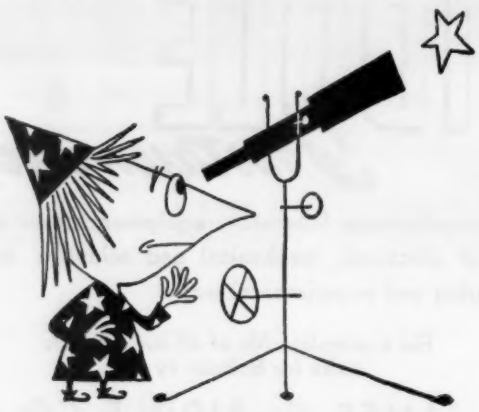

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Vol. 260 JULY, 1955 No. 1

AMERICAN AGRICULTURE—A GREAT PAST, A GREATER FUTURE *

BY

EZRA TAFT BENSON [1]

It is truly a double pleasure to appear before this joint meeting of the Philadelphia Society for Promoting Agriculture and The Franklin Institute. To visit this great historic city is to be deeply conscious of the lasting imprint of Benjamin Franklin's genius, of his priceless contributions to science, literature, philosophy and statesmanship.

The material evidence of these treasured gifts is present everywhere. Yet there is infinitely more. There is the spirit of Franklin's humanity which is as much a part of Philadelphia as Independence Hall. Here, it is what a man feels, even more than what he sees.

It is a signal honor to meet with an agricultural group which numbered Franklin among its long list of distinguished members and with The Franklin Institute which has performed such outstanding service in promoting his philosophy. He was among the first Americans to stress the vital importance of agricultural research and education. It is significant that today—two centuries later—great gains of agriculture continue to stem from these primary sources.

"Agriculture may diminish its labor and double its produce," Franklin wrote to Joseph Priestley. But even he could hardly have foreseen the production miracles of agriculture in this age.

In Franklin's time, we were preponderantly a rural people. The many who labored in the fields were able to produce only enough food and fiber for themselves and the few who followed other occupations. Today each of our 8.5 million agricultural workers turns out enough products to meet his own requirements, plus those of 18 additional persons. Nor does this comparison of productivity end even here. The American farm worker of today provides his customers with an

* Address presented at the joint meeting of the Philadelphia Society for Promoting Agriculture and The Franklin Institute, March 16, 1955, in the Lecture Hall of the Institute.

[1] Secretary of Agriculture, U. S. Department of Agriculture, Washington, D. C.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

infinitely more varied and nutritious diet than our Colonial forbears ever dreamed about.

We have only two-thirds as many people living on farms now as we had even half a century ago. They represent about 13.5 per cent of our 164 million population. Yet they are providing a greater abundance than ever before for all of us, plus some $3 billion worth of farm commodities for export each year. Even after these vast needs have been satisfied, personal experience qualifies me to testify that there is still some to spare for the government.

Every American shares in the unprecedented benefits which flow from our remarkable farm efficiency and productivity. Millions of workers who would be farmers in a less efficient agricultural economy have been released to other productive enterprises. They are in numerous professions and service industries. They are making the automobiles, refrigerators, television sets and the many other items which contribute to our better living standards.

We can best insure continuation of our great progress as a Nation by maintaining a free and fluid economy. We must not freeze people in any occupation. We must preserve our basic freedom to shift to new industries and new pursuits as demands change. A fundamental defect in the high, rigid farm price support program is that it tends to freeze agricultural production into fixed patterns without regard to shifting consumer preferences and demands.

Looking ahead, we may be reasonably certain of two things: first, that the decline in the percentage of Americans living and working on farms will continue, and, second, that our farmers will be called upon to produce an ever-increasing amount of food and fiber for our rapidly-growing urban population.

Census projections indicate that there may be 200 million Americans by 1970—an increase of some 36 million over today. During the same period, our farm population is unlikely to increase. It is less than 22 million now.

Our growth pattern as a Nation assures a bright future for farmers. During the next 15 years we will have to increase total agricultural output by approximately one-fourth to provide for each of our new citizens the same kinds and amounts of food available to the average person in the United States today. We must obtain this expanded production from roughly the same acreage in cultivation now. Even though we bring some new land into agricultural use through irrigation and reclamation each year, this gain is offset by the farm acreage which is lost as the population of the cities overflows into new suburban developments.

Here is a stirring challenge—and an unparalleled opportunity—for agriculture. Even more efficient utilization of our land and other resources will be required to meet the expanding markets of tomorrow.

We must press forward with soil fertility programs against the day when these reserves will be needed. Research and education must receive even increased emphasis as these twin tools lead the way of progress in agriculture.

Government can and will play a part in helping agriculture adapt to the new circumstances and conditions. By far the greater part of the job will be done, as it has in the past, by farmers themselves. It will be accomplished through the use of better breeds, feeds, seeds and generally improved farming practices. It will come about through research, education and market expansion and through further reduction in the time lag between the discovery of new methods and techniques and their application in the field. Maximum progress will come only if agriculture is free and farmers can make their own management decisions on their own farms with a very minimum of government regulation and control.

I am sure that Benjamin Franklin would be proud of America's great agricultural progress—progress which has come about in no small part along the lines he envisioned. I am equally certain that in this 20th Century he would see as many stimulating challenges on the farm as he saw in the 18th Century—perhaps even more, with our agricultural scientists taking their first, uncertain steps into a whole new realm of atomic possibilities.

As one of the most effective spokesmen of his age for the cause of world peace, Franklin today would certainly argue with all of his eloquence for the employment of atomic energy to improve the lot of mankind. He might well point an accusing finger at the men of the Kremlin and challenge them to raise up the living standards of their people that, in his ringing words, "men would cease to be wolves to one another." This failure of the Soviet system to provide for the basic needs of its own people could be one of the most important historical facts of our time.

One clear clue to the whole Russian riddle—made even more apparent with the fall of Malenkov last month—may be found in the shortcomings of agriculture under the collective farm system. Russia has not only failed to keep pace with farm progress in the free world. She has lagged far behind. This fundamental weakness may force internal changes which one day will provide a new basis for mutual understanding and lasting world peace.

Food, rather than nuclear weapons, could possibly become the decisive factor in shaping the course of history. Certainly at least a part of the Soviet sabre-rattling is designed to divert the attention of the Russian people from their own unhappy situation.

The masters of the Kremlin imply that shortages of food and consumer goods are the result of a capitalist conspiracy which forces Russia to devote so much of its production to defense of the homeland.

Yet the Russian peasants and workers may wonder how it is that the so-called "decadent" nations of the free world are able to mount this alleged threat of aggression against the Soviet Union—and still devote by far the greater part of their total production to improving the living standards of their people.

Close to three-fourths of Russia's industrial production by-passes the consumer. It goes into heavy industry to produce more heavy industry and armaments. In the United States, by contrast, nearly three-fourths of all production is for consumer use in the form of food, clothing, homes, automobiles, electrical appliances and many items which would be considered luxury goods in Russia.

The population of Russia is estimated at more than 200 million. This represents an increase of roughly one-third since 1928. Yet over this same period, Russia's cattle numbers have declined by about 8 per cent.

In the United States last year consumption of red meat reached 154 pounds per person. In addition, the average citizen here ate more than 28 pounds of chicken and turkey. For the Russian it was a different story. If he happened to be in the right lines at the right time, he might have been lucky enough to get nearly a pound of meat each week.

This would certainly have been less than he wanted but still perhaps as much as he could afford. The average Russian worker must labor two hours to earn enough to buy a single pound of beefsteak. An American industrial worker earns his pound of beef in 24 minutes—one-fifth of the time. The average American spends one-fourth of his disposable income for food—the average Russian one-half. If he were content to buy the same kinds and amounts of food as the Russian does, an American could get by for perhaps 10 to 15 per cent of his spendable income.

On Russia's 90,000 collective farms, there are roughly 50 million farm workers, about six times as many as in the United States. The American farm worker, as I mentioned earlier, turns out food and fiber for himself and 17 others. The Russian farm worker produces only enough for himself and 3 or 4 additional people. Perhaps he isn't doing quite that well, in view of the current official complaints.

Even these figures do not tell the full story. Our farmers provide their city customers with a broad variety of food, emphasizing heavily the nutritious, high protein items such as meat, dairy products, poultry and eggs and fresh fruits and vegetables. The Russian diet features bread, potatoes, cabbage soup and porridge. This is filling enough, to be sure. But it could get monotonous—even in a so-called "people's paradise."

The plight of the Russian farm worker is, in many ways, even more desperate than that of his city cousin. He lacks the machinery, the

fertilizer and the know-how required for a first-class production job. He is plagued by government inspectors and supervisors who assign him impossible quotas. If he manages to raise a few more bushels of wheat or a few more chickens, taxation and requisitioning policies deny him real benefit from his efforts.

Only after required deliveries are made to the government—and at niggardly prices—is provision made to meet his own needs. What he gets is mostly in kind and represents but a fraction of the fruits of his labor. This in large part he must eat to live. Such money as he receives, or is able to get from sale of produce he can ill spare on the so-called "free market," is of less value than even the money of urban dwellers. For the general shortage of consumer goods is especially prevalent in rural sections.

Now Russia proposes to solve food shortages through the creation of a corn-hog program much like the one which flourishes in Midwestern America today. The earlier plan to increase livestock production has fallen far short of its goal and there is good reason to believe that the projected expansion in hogs will be no more successful. Neither Russia, nor any other country for that matter, has the ideal combination of soil, climate and know-how which has made our Midwestern corn-hog belt the most productive area of its kind in the world. And you can't feed dialectical materialism to a hog.

I cannot envision a thriving or even moderately successful livestock industry operated by the state. I just can't see a Russian farm worker rolling out of his pallet at 3 A.M., fumbling for his boots on an earthen floor, and then rushing to the barn to supervise the farrowings of the state's sows. What is more important, I don't believe the Soviet peasant can see himself doing it, either.

This points up the basic weakness of Russian plans to expand her livestock industry. Somehow, Duroc Jerseys, Holstein-Friesians and even Rhode Island Reds just don't seem to mix with Marx and Engels.

The successful American dairyman knows that careful, yes even tender, handling of his herd is a most important factor in milk production. He takes great pride in his animals. He knows, moreover, that his livelihood depends upon efficient milk production.

If the Russian dairyman has a different outlook, it is quite understandable. He knows that what he milks from the cow will in turn be milked from him by the state.

Many factors doubtless entered into the recent shakeup in the Kremlin. Among them, however, agricultural problems certainly loomed large. Russian agriculture has been chronically ill for a long time. Shocking admissions of past failures were publicly aired within a few months of Stalin's death. The list of Beria's crimes on the occasion of his liquidation in the summer of 1953 included charges that he opposed measures necessary for the improvement of agriculture.

And Malenkov's ouster as Premier last month was accompanied by his "admission" of "guilt" for the agricultural situation. Attribution of responsibility for agricultural difficulties to these two individuals does not mean that they were in fact the ones at fault. Khrushchev actually had a far greater hand in agricultural policy over the years than Malenkov. But the care exercised to make first Beria and then Malenkov scapegoats for failure in this field is indicative of the magnitude of the agricultural problem.

It is not for me to speculate upon whether these happenings presage a new direction of Soviet policy—a lesser or greater threat of war. Conceivably a power-mad ruling clique could plunge the world into war in a last, desperate attempt to retain authority. We have, however, no indications that the Soviet situation has reached, or is approaching such a stage. We know that the men in the Kremlin face serious economic problems and that they are probably still torn by personality and power conflicts. But we have no reason to believe that the situation has suddenly become so acute that they are any more anxious than their predecessors to take the last desperate gamble.

The Malenkov purge, coming as it did after the frank acknowledgment of September 1953 of past agricultural failures, spotlighted again the fact that agriculture may be Russia's Achilles' heel. It has underscored the most candid admission of this weakness ever to come from Soviet leaders. It has also provided the Russian people with a clearer understanding that their own administrators, rather than the free world, have been responsible for the breakdown in food production.

Even in a complete dictatorship there must exist opinion, repressed though it may be by fear. The secret police may execute a man for what he says, but even they cannot successfully probe the inner recesses of a man's mind.

Somehow I cannot believe that it is possible to keep a people in complete bondage forever—though this conclusion may be based more upon hope than solid evidence. While it may be true that a peasant or a factory worker cannot revolt against a tank, it is also true that the ordinary people of Russia man the Soviet Army's tanks. It seems to me that the Russian dictatorship must one day face even greater internal stresses unless it embarks upon a program which will provide the people with tolerable living standards and at least some semblance of individual freedom.

If Russian planning and production were directed toward improving the living standards of the people instead of preparation for war, it would indeed be a happy day not only for that nation but for all of the world. More and better food, more consumer goods and hope for an even better future could ease the terrific economic and social pressures which must exist within the Soviet Union.

Meanwhile, we must keep America and the free world strong—

militarily strong, economically strong and, let us never forget, spiritually strong. Because the final decision on this great, overriding question of war or peace may not be ours to make, we must be prepared for any eventuality.

The maintenance of an expanding, prosperous and free agriculture in our own country is essential to the security of the United States. I would place special emphasis upon a free agriculture. Freedom makes for progress. Without freedom our own farmers would be unlikely to fare much better than those of the Soviet Union. Certainly the example of Russia should be convincing evidence that the withering hand of totalitarian control blights all of agriculture.

As Thomas Jefferson once wrote: "Were we directed from Washington when to sow and when to reap, we should soon want bread."

Perhaps this quotation has come to the attention of the Russian leadership. At any rate, only last week it was announced that the practice of handing down planting and production decrees from Moscow would be modified to permit a much greater voice in such decisions by farmers and farm managers.

If the Russian government has done nothing else for the free world, it has at least provided a laboratory for testing nearly every conceivable means of state intervention in agriculture. It has demonstrated for all to see that farming will not flourish in a climate which denies real incentives to farmers.

It has proved beyond a doubt that complete regimentation stifles agricultural production. It has demonstrated that the agricultural system it seeks to impose on other countries as a part of the communist yoke yields only chronic shortages. It has unwittingly provided the entire world with a most unfavorable comparison of the merits of communism versus private enterprise in the field of food production.

There is a moral in all of this for America. Farmers in this country have had ample opportunity to try on for size the straitjacket of production and price controls and marketing restrictions. Most of them didn't like the fit.

The long-range objective of this Administration's farm program is to bring about greater stability of income, better balanced production and, with this, greater freedom for farmers to make their own management decisions. I am confident that we will succeed.

It is important to remember that the flexible price support provisions of the new law do not become effective until this year's harvest. Our current problems have come about entirely under the old system of high, rigid, war-time price supports.

For the year ahead, I believe the outlook is generally good—both for agriculture and the Nation as a whole. Business activity is at the highest peacetime level in our history. This assures a continuing

strong domestic demand for the products of our farms. Agricultural exports are likely to be at least 10 per cent higher this year than in 1954.

Net farm income in 1955 is expected to be near last year's total of $12 billion. While net farm income has declined in every year but one since 1947, it is important to remember that this same income is being shared by fewer farmers.

Actually, per capita net income of farmers has been steadily advancing, although not as rapidly as for most other segments of our population. For example, although net income of farm people from all sources declined 3 per cent from 1953 to 1954, the farm population was down 3.5 per cent. Thus, there was a slight rise in net per capita income. Between 1947 and 1954, while total net realized farm income was declining by some 25 per cent, per capita income of farm people from all sources rose by 6 per cent. That is not enough, however. The farmer deserves to share fully and fairly in this Nation's prosperity.

There is another phase of the farm situation which deserves attention. Commodity Credit Corporation today either owns outright or has under loan approximately $7.4 billion worth of farm commodities. This represents one-fourth of the value of our total agricultural production for an entire year. Until some of these surpluses built up under past price support programs are reduced substantially, agriculture will not be without its price problems. We are paying now the cost of past borrowing against future markets.

I share fully the view expressed by President Eisenhower in his State of the Union Message of this year, when he said: "Agricultural programs have been redirected toward better balance, greater stability and sustained prosperity. We are headed in the right direction. I urgently recommend to the Congress that we continue resolutely on this road."

The great forward progress of American agriculture through the years far over-shadows its transitory problems. Truly American agriculture may point to a great past. I am convinced that its future will be even brighter—as bright as we are determined to make it, with the help of God.

THE FOLDED TREE *

BY

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DON W. WARREN [1] AND JESSE B. WRIGHT [1]

Part I **

1. INTRODUCTION

The problem of constructing circuits which perform a certain function and have some formal properties contributing to engineering efficiency was considered by Shannon (1).[3] Part of Shannon's problem (Part II of (1), pp. 81–89) was to formulate an arithmetic condition such that for any sequence of positive integers satisfying that condition a tree circuit can be constructed whose load distribution is given by that sequence. That condition is identical with our notion of "admissibility," defined in Section 3 of this paper.

The present paper takes up this same problem, but is entirely self-contained. No reference to Shannon's paper is necessary either for definitions of concepts or for proofs of theorems. Our results go beyond those of Shannon, both in that we prove the necessity of the admissibility condition and also in that we give a constructive technique, helpful to a practical engineer, for constructing a folded tree with any given load distribution satisfying the condition of admissibility. To obtain these results we must formulate a precise definition of the term "folded tree." Although Shannon does not give a precise definition of any corresponding concept, it is clear that our precisely defined concept applies to the circuits which he considers.

We shall use some of the concepts of our previous paper (2), but we define them anew here, so no acquaintance with that paper is presupposed. Our method, here as in (2), is to discuss diagrams rather than circuits directly. The diagrams may be realized by circuits of various different kinds, for example, relay transfer contact nets discussed in (1), and electronic digital computing circuits. The advantages of this approach are (1) that the range of application of results is wider, and (2) that the problems, being abstract, can be solved within pure mathematics. Our methods and results are intimately connected with

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[3] The boldface numbers in parentheses refer to the references appended to Part I of this paper.

the field of mathematics known as the theory of linear graphs; but they are not an application of any previously known materials of that field, and consequently we presuppose no knowledge of it. Although our definitions and theorems concern diagrams, we shall frequently comment on their significance for physical circuits.

2. DEFINITION OF FOLDED TREE

In this section we formulate a precise definition of the term "folded tree" and show that the diagrams covered by this term are suitable for representing some familiar circuits constructed out of standard devices.

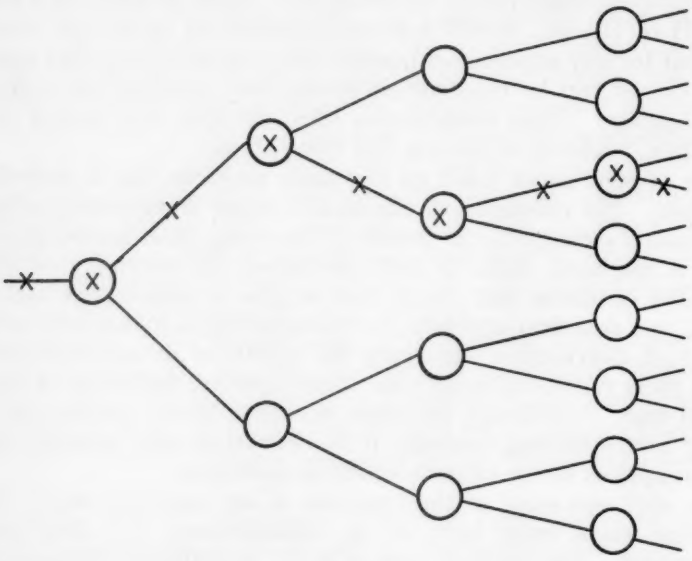

FIG. 1.

We shall be concerned with *vertex diagrams* which are arrangements of small circles and straight lines, such that (1) each circle has just three lines touching its circumference, one on its left and two on its right, (2) each end of each line may touch either a circle (as in Fig. 1) or the ends of any number of other lines (as in Fig. 10) or it may touch nothing (as in Fig. 1), and (3) no circles touch each other. (In this paper if a word is italicized in a sentence, then the word is defined in that sentence.) The circles are called *vertices* and the lines *wires*; the left-hand wire is called the *vertex-input*, the upper right-hand wire is called the *upper vertex-output*, and the lower right-hand wire, the *lower vertex-output*. The usage here of "output" and "input" is nearer to that of (1) than to (2).

The first vertex diagram we introduce is an *n-bay tree*, for which the following recursive definition is provided:

1. A *1-bay tree* consists of just one vertex with its input and output wires; this vertex is the *first* (and only) *bay* of the 1-bay tree;
2. An $(i + 1)$-*bay tree* results from an i-bay tree when, to each vertex-output u of a vertex in the ith bay, a new vertex is joined so that u is the vertex-input of the new vertex; the new vertices constitute the $(i + 1)$th bay;
3. An n-bay tree has only the vertices and wires provided for in 1 and 2; no diagrams other than those so provided for are trees.

The *input* of an n-bay tree is the vertex-input of the vertex of the first bay. An *output* of an n-bay tree is a vertex-output of a vertex in the nth bay. Note that the tree input is the only wire in the tree which is not a vertex-output, and that the tree outputs are the only wires which are not vertex-inputs. Note also that there are 2^n outputs of an n-bay tree. The result of deleting the crosses (but no lines or circles) from Fig. 1 is a 4-bay tree. (Note that each bay in this figure is simply a vertical column of vertices. This will be true of all figures of trees in this paper, although the definition of "tree" does not require this.)

A wire is to be understood as being at any one moment in either of two *states*, state 1 or state 0, and a vertex as being in just one of two *settings*, an upper setting or a lower setting. The state of a vertex-output is determined by the setting of the vertex and the state of the vertex-input: if the vertex-input is in state 1 and if the vertex is in the upper setting, then the upper vertex-output is in state 1 and the lower vertex-output in state 0; if the vertex-input is in state 1 and the vertex in the lower setting, then the upper vertex-output is in state 0 and the lower vertex-output in state 1; if the vertex-input is in state 0, then regardless of the setting of the vertex the state of both vertex-outputs is 0.

Before proceeding we will relate trees to the net diagrams of (2) and described two physical realizations of trees. In the net diagram of Fig. 2 each square is a conjunction ("and") element whose output is in state 1 if and only if both its inputs are in state 1. A vertex, together with its vertex-input and vertex-outputs, and the net of Fig. 2 represent the same type of circuit if exactly one of the input pair b,b' is in state 1 at any one time. (The use of the word "input" in this connection is similar to the use in (2); it differs significantly from the use of the word in the major part of this paper.) Wire a of Fig. 2 is the vertex-input, and wires c and d are the vertex-outputs, and the vertex is in the upper setting just in case b is in state 1. Nets like the one of Fig. 2 may clearly be combined to form trees.

A conjunction element may be realized by a crystal diode or vacuum tube circuit whose output voltage is high (state 1) whenever both circuit inputs are at high voltage, otherwise the circuit output voltage is low (state 0). Two such circuits, interconnected as in Fig. 2, constitute a physical interpretation of a vertex with its vertex input and two vertex-outputs. Wire c is in state 1 whenever a and b are both in state 1, and d is in state 1 whenever a and b' are both in state 1.

A vertex may also be realized by a relay transfer element. A relay transfer element is a (mechanical) single-pole double-throw switch with the wire of the pole realizing the vertex-input and the wires from the output contacts realizing the vertex-outputs. The transfer element

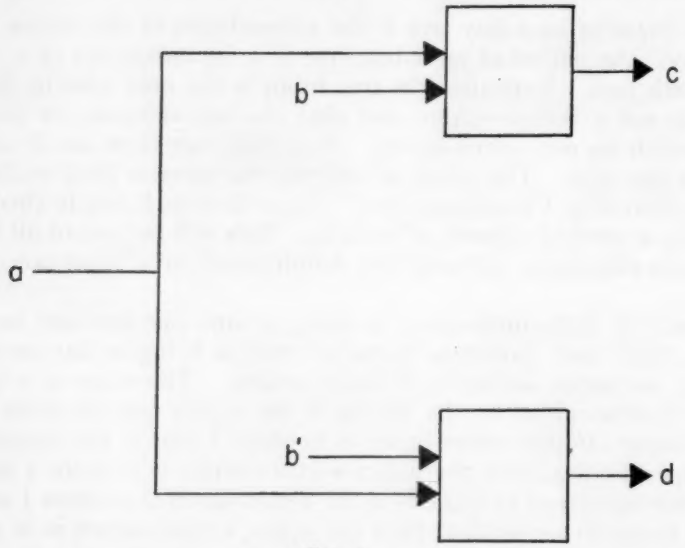

FIG. 2.

is controlled by a magnet and spring working in opposition, the two positions being represented by the two vertex settings. Thus, if current is flowing (state 1) in the vertex-input, it flows in the upper vertex-output whenever the relay is not activated and in the lower vertex-output whenever the relay is activated.

It should be noted that our vertex diagrams (trees and the diagrams of Section 6) are particularly suited to represent relay contact nets composed of transfer elements, namely, single-pole double-throw switches, electro-magnetically controlled. In such electrical nets the wires to the device (magnet) which controls the settings of the transfer contacts are not connected into the contact net itself. That our vertex diagrams do not show the wires which determine the vertex settings is intended to represent that feature. In contrast the wires b,b' of an electronic circuit of the sort represented by Fig. 2 are, in

general, like a, c, and d, connected to other components of the circuit. Electronic circuits can thereby realize nets which are more complex than vertex diagrams. Hence, the class of circuits represented by vertex diagrams (in the broad sense of Section 6) is narrower than the class of circuits represented by diagrams constructed from conjunction elements. It turns out, for example, that in a certain sense of "cost" the tree is probably a minimal diagram in the first class, while it is not at all minimal in the second class (see Section 6). To put the point alternatively, a generally efficient way to do complete decoding with relays is by means of a tree, but that is not generally an efficient way to do complete decoding with vacuum tubes or crystal diodes.

A *chain* of an n-bay tree is a sequence $X_1, \ldots, X_{2n+1}$, where X_1 is the tree input and where, for $i \leqq 2n$, if X_i is a vertex-input of some vertex, then X_{i+1} is that vertex and, if X_i is a vertex, then X_{i+1} is one of its vertex-outputs. It follows that X_{2n+1} is a tree output. In

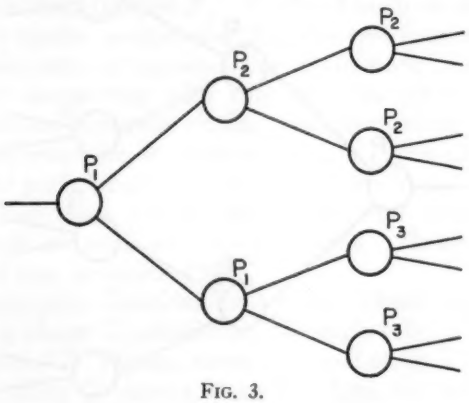

FIG. 3.

Fig. 1 the vertices and wires marked with crosses constitute a chain. It is obvious that each tree output is a member of one and only one chain.

In an n-bay tree the settings of the vertices and the state of the tree input vary independently of each other, but once these are determined the state of each wire is determined. If the tree input is in state 0, then every wire is in state 0, regardless of the settings of the vertices. If the tree input is in state 1, then there will be a chain whose every wire is in state 1 and such that every wire not on the chain is in state 0. What chain it is will depend on the settings of the vertices, since the state of each vertex-output is determined by the setting of the vertex and the state of its vertex-input.

We are interested in trees in which the settings of the vertices do not vary completely independently of each other, but whose vertices are partitioned into a number of classes, all the vertices of each class being in the same setting at any one time. In such trees we indicate the

class to which each vertex belongs by affixing the same label to every vertex of a given class, vertices which belong to different classes being given different labels. We call such a tree a "labeled-tree," and define it as follows. An *n-bay labeled-tree* is formed from an n-bay tree by marking each vertex with exactly one label from a set of m labels, and using each of the m labels to mark at least one vertex; m can be less than, equal to, or greater than n. We require, of course, that all vertices having the same label be in the same setting. Figure 3 is a 3-bay labeled-tree in which $m = n$.

An *n-bay folded tree* is an n-bay labeled-tree in which there are exactly n labels (that is, $m = n$) and, for every chain and for each label, there is at least one vertex with that label in the chain. It follows that there is exactly one vertex in each chain with a given label, since there

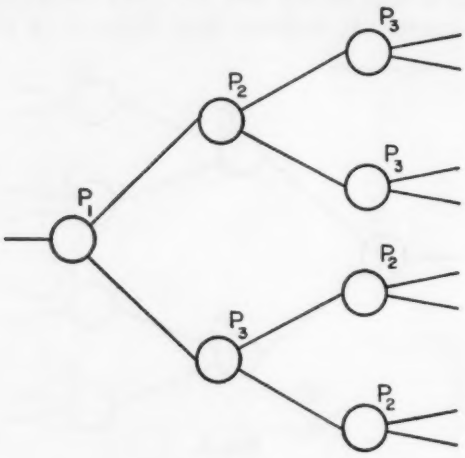

FIG. 4.

are n labels and n vertices in a chain. A remark on the appropriateness of the term "folded" in this connection will be made in the Appendix. Note that Fig. 4 is a folded tree, but Fig. 3 is not.

Folded trees are important among labeled-trees because they can function as "complete decoding nets" in the sense given in (2). We must explain what this means and why it is so. A circuit represented by a labeled-tree performs its function when the tree input is in state 1 (for example, after all the transfer contacts of a relay contact net are set, an electrical signal introduced at the tree input will emerge at the desired output). We define a *tree state* of a labeled-tree to be a definite assignment of its vertex settings such that all vertices with the same label are set the same, and such that the tree input is in state 1. It is clear that there are 2^m different tree states for a labeled-tree having m distinct labels.

We say that an n-bay labeled-tree is *complete decoding* if and only if the number of labels m is equal to n, and for each tree state there is at least one tree output which is in state 1 in that tree state and state 0 in every other tree state. It follows that there is only one tree output in state 1 in any tree state of a complete decoding n-bay labeled-tree, since it has exactly 2^n tree states and 2^n tree outputs. (A nearly identical concept of complete decoding is discussed in Section 3 of (2).)

Theorem 1: A necessary and sufficient condition for a labeled-tree to be complete decoding is that it be a folded tree.

Proof of necessity: Consider an n-bay labeled-tree which is complete decoding. Consider any tree state S and the tree output Q which is in state 1 only in S. There is only one chain C which includes Q. Suppose that, for some label P_i, there is no vertex in C labeled P_i. Now let S' be a different tree state which coincides with S except for the settings of the vertices labeled P_i. Q would be in state 1 in S' as well as in S, contrary to the assumption that the tree is complete decoding. Hence, every tree state determines one chain such that for each label there is at least one vertex with that label in the chain. Different tree states determine different chains; and since there are 2^n different tree states, there must be 2^n different chains, each of which has the property that for each label there is at least one vertex with that label in the chain. But these are all the chains that there are, since an n-bay labeled-tree contains exactly 2^n chains. Hence, any complete decoding labeled-tree is a folded tree.

Proof of sufficiency: Consider any n-bay folded tree. Every tree output Q is on a chain C containing, for each label P_i, one and only one vertex labeled P_i. Hence, there is a tree state S in which all the wires of C (including Q) are in state 1. Also, for any different tree state S', there will be at least one vertex of C in a different setting and so Q will be in state 0 in S'. Therefore, (1) each tree output will be in state 1 in one and only one tree state. Furthermore, (2) any two tree outputs will be in state 1 in different tree states, which can be proved as follows. If Q_1 and Q_2 are different tree outputs on chains C_1 and C_2, respectively, then let i be the number of the latest bay in which C_1 and C_2 have a vertex V in common. Suppose that V is labeled P_k, and suppose (without loss of generality) that C_1 includes the upper vertex-output of V. C_2 must then include the lower vertex-output of V. In order for Q_1 to be in state 1, V will have to be in the upper setting, and, in order for Q_2 to be in state 1, V will have to be in the lower setting. Hence, Q_1 and Q_2 are in state 1 in different tree states.

Since there are as many tree states as tree outputs it follows from (1) and (2) that the tree is complete decoding. This completes our proof of Theorem 1.

An *n-bay standard tree* is an n-bay labeled-tree in which any two vertices have the same label if and only if they are in the same bay. Figure 5 is a 3-bay standard tree. (If we interpret the vertices as in Fig. 2, an n-bay standard tree is essentially the same as the 2-n tree defined in Section 4 of (2).)

Consider now two electrical devices, T_1 realizing a 6-bay standard tree and T_2 realizing a 6-bay folded tree in which the number of vertices labeled $P_1, \ldots, P_6$, respectively, is 1, 12, 12, 12, 13, 13. Both perform the same decoding function but the latter has a more nearly equal distribution of labels. Since all of the vertices with the same label are set in the same way at any one time, this means that the load distribution on the wires controlling the settings of the vertices is

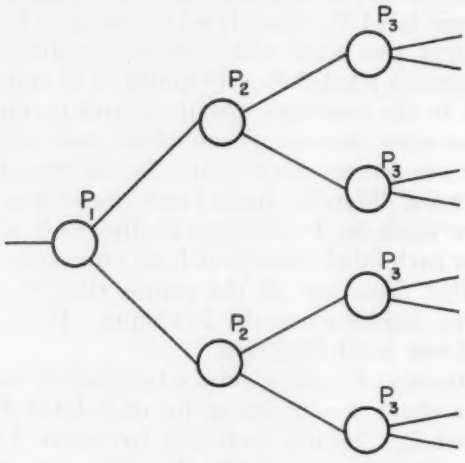

FIG. 5.

more nearly equal in T_2 than in T_1. (The sequence 1, 12, 12, 12, 13, 13 will later be referred to as the "loading sequence" of the tree realized by T_2.) If the devices are constructed of relays (and for reasons indicated earlier in this section the tree is more important for relay contact nets than for electronic digital computing circuits), a further factor needs to be considered. This factor is the number of contacts that each available coil controls. One transfer contact can realize one vertex, but two contacts of the same relay cannot realize vertices with different labels. If one had available relays with eight transfer contacts each, T_2 with load distribution 1, 12, 12, 12, 13, 13 would require eleven relays, while the (standard tree) circuit T_1 would require only ten relays. On the other hand, a (folded tree) circuit with load distribution 1, 8, 8, 15, 15, 16 would require only nine relays. Depending upon what physical equipment is available, different load distributions

for a folded tree offer greater or less advantage. It is therefore of practical interest to know what different load distributions are possible for folded trees.

We are thus led to consider the interesting problem of determining how the P_i's can, in general, be distributed among the $2^n - 1$ vertices of a folded tree. More precisely, given a sequence of n integers $a_1, \ldots, a_n$, is there an n-bay folded tree having, for each i, a_i vertices labeled P_i? A condition for a sequence of positive integers to have a folded tree corresponding to it was stated in (1) and there proved to be sufficient. That condition is the property of "admissibility" as defined in the following section. All the sequences discussed in the preceding examples have this property; 1, 2, 5, 6, 20, 29 is a sequence that does not. In Section 3 of this paper we prove admissibility to be a necessary condition also; and in Sections 4 and 5 we present and justify a method of construction which when applied to any admissible sequence of positive integers results in a folded tree corresponding to that sequence, thus providing a proof of sufficiency different from Shannon's earlier proof which was not constructive.

3. LOADING SEQUENCES; ADMISSIBILITY

In this section we define an arithmetic condition on sequences of non-negative integers, which we shall call the condition of "admissibility," and show it to be a necessary condition for a sequence to represent a distribution of labels among the vertices of a folded tree. To define "admissibility" we must first introduce some additional notions.

Let $V(i,j)$, $1 \leqq j \leqq 2^{i-1}$, be the jth vertex in the ith bay. Inasmuch as the bays are vertical in our figures, $V(i,1)$ is the top vertex of the ith bay, $V(i,2)$ the vertex next to the top, etc. It is clear that the vertex-outputs of $V(i,j)$ are the vertex-inputs of $V(i+1,2j-1)$ and $V(i+1,2j)$. Now let us consider all the chains which include $V(i,j)$. These all have a common vertex in each of the first i bays. That part of the labeled-tree which includes $V(i,j)$ and all vertices from the last $n - i$ bays which are on chains containing $V(i,j)$, together with all vertex-inputs and vertex-outputs of those vertices, is itself a labeled-tree and is called a *minor tree*. We refer to it as $T(i,j)$ since it is determined by $V(i,j)$. $T(1,1)$ is, of course, the original labeled-tree itself. Note Fig. 6 in which $T(2,2)$ is circled.

We assume without any loss of generality that the m labels of $T(1,1)$ are $P_1, P_2, \ldots, P_m$. The *index* of P_k in $T(i,j)$ is the number of vertices labeled P_k in $T(i,j)$ which we denote by $a_k(i,j)$. The *loading sequence* $S(i,j)$ of $T(i,j)$ is the sequence $a_1(i,j), \ldots, a_m(i,j)$. Note that $a_k(i,j)$ is sometimes 0. $S(1,1)$ is then the loading sequence of the original labeled-tree $T(1,1)$. In Fig. 6, $S(2,2)$ is 2, 2, 1, 2, while $S(1,1)$ is 3, 5, 3, 4. Note that $T(i,j)$, $V(i,j)$, $S(i,j)$, and $a_k(i,j)$ are functionally dependent upon the labeled-tree under consideration as well as upon

i and j, even though that functional dependence is not explicit in our notation.

Where S is a finite sequence of non-negative integers $a_1, \ldots, a_m$, we define $M(S)$ to be the sequence rearranged in monotonic non-decreasing order, zeros deleted. Thus, if S is 3, 7, 5, 0, 2, 1, 0, 5, $M(S)$ is 1, 2, 3, 5, 5, 7.

A sequence S of m integers satisfies the *unit condition* if there is a 1 in the sequence. S satisfies the *total sum condition* if the sum of the terms of S is $2^p - 1$, where p is the number of nonzero terms. S

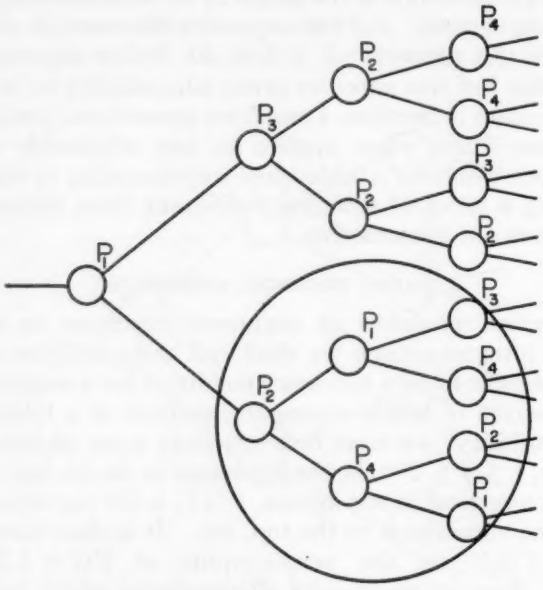

FIG. 6.

satisfies the *partial sum condition* if, for each $k \leqq p$, the sum of the first k terms of $M(S)$ is $\geqq 2^k - 1$. A sequence S is *admissible* if it satisfies all of these three conditions. Thus, 0, 5, 0, 1, 0, 5, 4 is an admissible sequence in which $m = 7$ and $p = 4$; and 1, 9, 5, 10, 6 is an admissible sequence in which $m = p = 5$.

Theorem 2: A minor tree $T(i,j)$ of an n-bay folded tree is an $(n - i + 1)$-bay folded tree.

Proof: Obviously $T(i,j)$ has $n - i + 1$ bays. All the chains of $T(1,1)$ containing $V(i,j)$ have the vertices of the first i bays in common. These are labeled with exactly i of the labels. Now each chain must contain each of the remaining $n - i$ labels in the last $n - i$ bays. But these chains (with the vertices of the first $i - 1$ bays deleted) are the chains of $T(i,j)$. Hence, $T(i,j)$ is a folded tree.

Theorem 3: In an n-bay folded tree $S(i,j)$ has $i - 1$ zeros and $n - i + 1$ non-zero terms.

The proof is immediate from Theorem 2.

Theorem 4: In a folded tree, if $V(i,j)$ is labeled P_k, then $a_k(i,j) = 1$.

Proof: Since $V(i,j)$ is labeled P_k, no other vertex in any chain containing $V(i,j)$ is labeled P_k. For suppose that $V(i',j')$, $i' \neq i$, on a chain C with $V(i,j)$ is also labeled P_k. Then C has $n - 2$ other vertices which must (in order to satisfy the folded tree condition) be labeled with $n - 1$ other labels which is impossible.

Theorem 5: In a folded tree $S(i,j)$ satisfies the unit condition.

Proof: This follows directly from Theorem 4.

Theorem 6: In a folded tree $S(i,j)$ satisfies the total sum condition.

Proof: This follows from Theorem 3 since there are $2^{n-i+1} - 1$ vertices in $T(i,j)$.

Theorem 7: In an n-bay folded tree $a_k(i + 1,2j - 1) = 0$ if and only if $a_k(i + 1,2j) = 0$.

Proof: $T(i,j)$ by Theorem 2 is a folded tree and, therefore, each P_k which labels any vertex of $T(i,j)$ has to label at least one vertex in each chain of $T(i,j)$. Every such chain contains $V(i,j)$. Beyond $V(i,j)$ each chain of $T(i,j)$ lies wholly within $T(i + 1,2j - 1)$ or $T(i + 1,2j)$. Hence any label occurs in $T(i + 1,2j - 1)$ if and only if it occurs in $T(i + 1,2j)$, from which Theorem 7 follows directly.

Theorem 8: In an n-bay folded tree $S(i,j)$ satisfies the partial sum condition.

Proof: The proof is by induction, beginning at the nth bay of the tree and going to the left. We first show (1) that, for every $j \leqq 2^{n-1}$, the theorem holds for $S(n,j)$. We then show (2) that, if it holds for $S(i + 1,j)$ for every $j \leqq 2^i$, it holds for $S(i,j)$ for every $j \leqq 2^{i-1}$. (1) is true by Theorem 3 for $S(n,j)$ has only one non-zero term. We prove (2) by showing that, if it holds for $S(i + 1,2j - 1)$ and S$(i + 1,2j)$, then it holds for $S(i,j)$. Suppose $V(i,j)$ is labeled P_q. Let $M(S(i,j))$ be $a_q(i,j)$, $a_{q_1}(i,j)$. $a_{q_2}(i,j)$, . . . , $a_{q_{n-i}}(i,j)$, where, of course, $a_q(i,j) = 1$. By hypothesis $S(i + 1,2j - 1)$ and $S(i + 1,2j)$ satisfy the partial sum condition; in other words, for each $k \leqq n - i$, the sum of the first k terms of $M(S(i + 1,2j - 1))$ or $M(S(i + 1,2j)) \geqq 2^k - 1$. Now, if such a condition holds for a monotonic non-decreasing sequence, it holds for the sequence in any order. Furthermore, since, for $k \neq q$, $a_k(i,j) = a_k(i + 1,2j - 1) + a_k(i + 1,2j)$, with the aid of Theorem 7 it is

easy to see that the non-zero terms of $S(i+1,2j-1)$ are $a_{g_1}(i+1, 2j-1), \ldots, a_{g_{n-i}}(i+1,2j-1)$ and the non-zero terms of $S(i+1, 2j)$ are $a_{g_1}(i+1,2j), \ldots, a_{g_{n-i}}(i+1,2j)$. Hence for each $k \leqq n-i$, $\sum_{x=1}^{k} a_{g_x}(i+1,2j-1) \geqq 2^k - 1$ and $\sum_{x=1}^{k} a_{g_x}(i+1,2j) \geqq 2^k - 1$. Hence, $\sum_{x=1}^{k} (a_{g_x}(i+1,2j-1) + a_{g_x}(i+1,2j)) \geqq 2^{k+1} - 2$. Clearly,

$$a_q(i,j) + \sum_{x=1}^{k} a_{g_x}(i,j) = 1 + \sum_{x=1}^{k} (a_{g_x}(i+1,2j-1) + a_{g_x}(i+1,2j));$$

hence for any $k \leqq n-i$, $a_q(i,j) + \sum_{x=1}^{k} a_{g_x}(i,j) \geqq 2^{k+1} - 1$, which means that the sum of the first $k+1$ terms of $M(S(i,j))$ is $\geqq 2^{k+1} - 1$, proving that $S(i,j)$ satisfies the partial sum condition.

Theorem 9: The loading sequence $S(1,1)$ of an n-bay folded tree is an admissible sequence of n non-zero terms.

This result follows immediately from Theorems 3, 5, 6, and 8, putting $i = j = 1$.

4. THE SPLITTING TECHNIQUE

In this section we provide an effective method of constructing a folded tree with a given admissible loading sequence of positive integers. The proof that this method will produce a folded tree is given in the next section. The procedure consists in taking the loading sequence proposed for the folded tree $T(1,1)$, selecting the unit term for $V(1,1)$, and dividing the remaining terms so as to yield two admissible sequences, one for $T(2,1)$ and the other for $T(2,2)$. This procedure, called the "splitting technique," is then repeated for each of those two sequences to provide admissible sequences for $T(3,1)$, $T(3,2)$, $T(3,3)$, and $T(3,4)$. The process is iterated until $T(n,2^{n-1})$ is reached. It will be proved in the next section that each vertex is thus provided with a label and that, for each k, the proper number of vertices will be labeled P_k.

It must be admitted that there are alternative splitting techniques which applied to the same admissible sequence give different folded trees. Our splitting technique, for example, applied to the admissible sequence 1, 4, 5, 5 gives a folded tree having three different labels in its last bay, but there is also a folded tree with the same loading sequence which has only two different labels in its last bay (see Fig. 7).

In this section and in Section 5 we let $S'(i,j)$, which is $a_1'(i,j)$, $a_2'(i,j), \cdots, a_n'(i,j)$, be the sequence obtained by repeated applications of the splitting technique from $S'(1,1)$, which is an arbitrary admissible

sequence without zero terms. The symbols $S'(i,j)$ and $S(i,j)$ are to be clearly distinguished even though in some cases they refer to the same sequence of numbers. $S'(i,j)$ is the sequence derived by our splitting technique from a given admissible sequence $S'(1,1)$, whereas $S(i,j)$ is the loading sequence of $T(i,j)$, a minor tree of a given labeled-tree.

Let us suppose that $S'(i,j)$ is a given admissible sequence. We describe now the *splitting technique* which determines $S'(i+1,2j-1)$ and $S'(i+1,2j)$. For every k, if $a_k'(i,j) = 1$ or 0, then let $a_k'(i+1,2j-1) = a_k'(i+1,2j) = 0$. Let $M(S'(i,j))$ be $1,d_1,\cdots,d_p$. (It will turn out that $p = n - i$.) We then determine sequences $b_1,\cdots,b_p$ and $c_1,\cdots,c_p$, such that, if d_x is $a_k'(i,j)$, b_x and c_x are to be $a_k'(i+1,2j-1)$ and $a_k'(i+1,2j)$, respectively. The recursive procedure for determining b_x and c_x is as follows: (1) put $c_1 = 1$ and

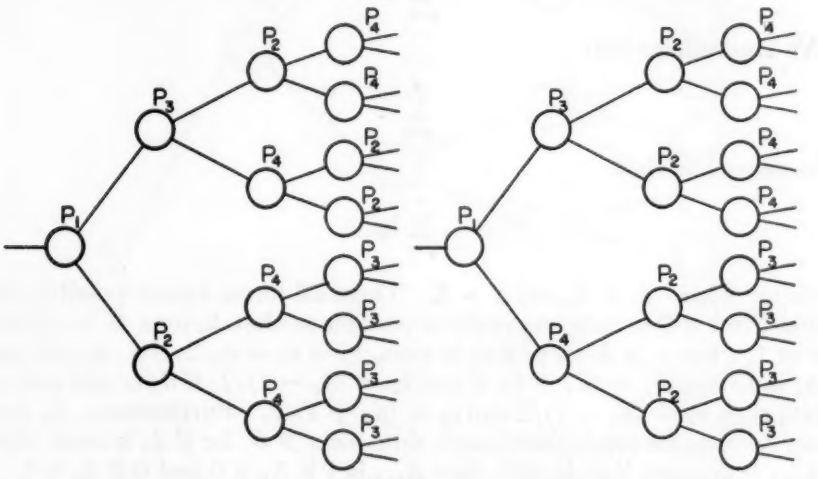

FIG. 7.

$b_1 = d_1 - 1$; (2) if $b_1 = 1$ (that is, if $d_1 = 2$), proceed to (3), otherwise first put $b_2 = 1$ and $c_2 = d_2 - 1$; (3) (for each x, $x \geqq 3$, and, for $x = 2$ if $d_1 = 2$) assuming that $b_1,\cdots,b_{x-1}$ and $c_1,\cdots,c_{x-1}$ have been determined, put

$$b_x = \left[\frac{d_x + \Delta_x}{2}\right]$$

and

$$c_x = \left[\frac{d_x + 1 - \Delta_x}{2}\right],$$

where

$$\Delta_x = \sum_{y=1}^{x-1} (c_y - b_y).$$

Here $[\phi]$, for any real number ϕ, is the integral part of ϕ, that is, the greatest integer not greater than ϕ. The reader can readily verify that for each x, $b_x + c_x = d_x$.

In actual calculation the work is even simpler than it appears from the formal statement of the procedure. The idea behind the splitting technique is simply to provide for the 1 in each sequence, and, for each d_x, to find a b_x and c_x such that $b_x + c_x = d_x$, in such a way that

$$\sum_{y=1}^{x} b_y$$

is as nearly equal as possible to

$$\sum_{y=1}^{x} c_y.$$

We also require that

$$\sum_{y=1}^{x} c_y$$

be never less than

$$\sum_{y=1}^{x} b_y,$$

except where $d_1 > 2$ and $x = 2$. Thus, all terms except possibly the first three will be split as evenly as possible so that Δ_x for $x > 3$ is either 0 or 1. For $x > 3$, then, if d_x is even, $b_x = c_x = d_x/2$; if d_x is odd and Δ_x is 1, then $b_x = (d_x + 1)/2$ and $c_x = (d_x - 1)/2$; if d_x is odd and Δ_x is 0, then $b_x = (d_x - 1)/2$ and $c_x = (d_x + 1)/2$. Furthermore, Δ_x does not have to be recalculated each time for $x > 3$; for if d_x is even, then $\Delta_{x+1} = \Delta_x$, and if d_x is odd, then Δ_{x+1} is 1 if Δ_x is 0 and 0 if Δ_x is 1.

The following table carries through a calculation from $S'(i,j)$ to $S'(i + 1,2j - 1)$ and $S'(i + 1,2j)$.

$S'(i,j)$	0	3	0	1	8	14	5
d-sequence	3	5	8	14			
b-sequence	2	1	5	7			
c-sequence	1	4	3	7			
$S'(i + 1,2j - 1)$	0	2	0	0	5	7	1
$S'(i + 1,2j)$	0	1	0	0	3	7	4

It is simple to rearrange the terms of the b and c sequences in forming the sequences $S'(i + 1,2j - 1)$ and $S'(i + 1,2j)$ in accordance with the rule that where d_x is $a_k'(i,j)$, b_x and c_x are $a_k'(i + 1,2j - 1)$ and $a_k'(i + 1,2j)$, respectively.

To illustrate the case where $d_1 = 2$, consider the $M(S'(i,j))$ sequence 1, 2, 5, 9, 14. Here the b-sequence is 1, 2, 5, 7 and the c-sequence is 1, 3, 4, 7.

Since we have shown how an admissible $S'(i,j)$ is split into $S'(i+1,2j-1)$ and $S'(i+1,2j)$, it is easy to see that given $S'(1,1)$ all the $S'(i,j)$ can be calculated, provided all the applications of the

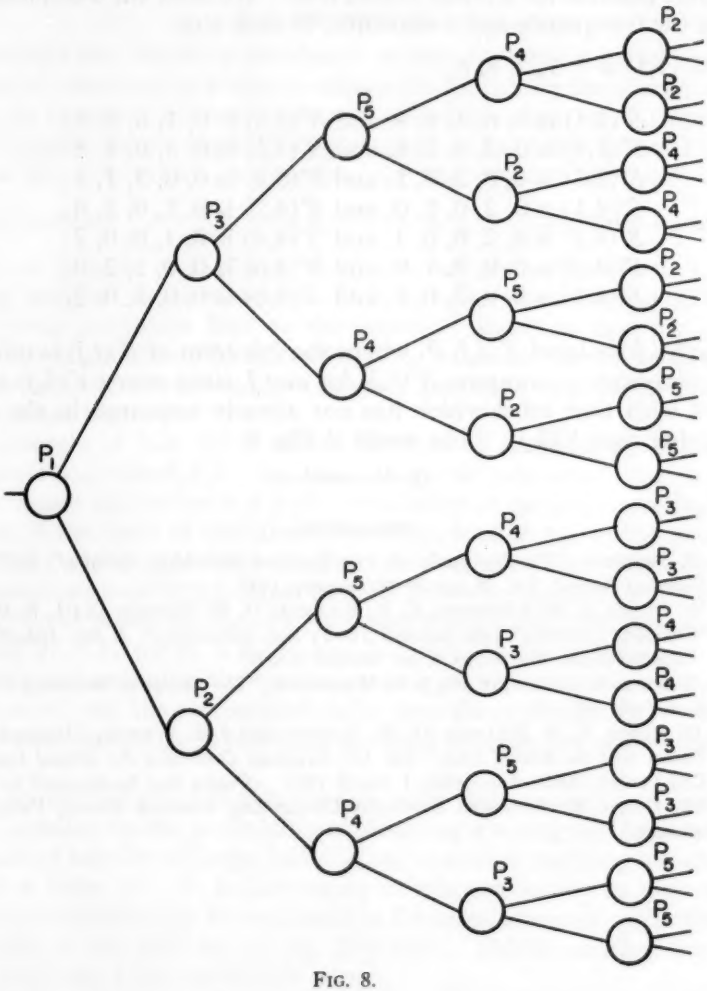

FIG. 8.

splitting technique yield admissible sequences. Once all these have been calculated, a folded tree whose $S(i,j)$ is $S'(i,j)$ can easily be constructed. If there are n terms in $S'(1,1)$, then, of course, we want an n-bay folded tree. An n-bay tree is constructed and each vertex

$V(i,j)$ is labeled P_k where $a_k'(i,j)$ is unity. (In the next section we prove that a vertex will be assigned one and only one label by this procedure.)

We will now provide an example showing the use of this procedure in constructing a 5-bay folded tree with the given admissible loading sequence, 1, 7, 7, 8, 8, which is one of the most evenly balanced loading sequences possible for a 5-bay folded tree. We omit the d-sequence, as well as the b-sequence and c-sequence, in each step.

Since $S'(1,1)$ is 1, 7, 7, 8, 8,

$S'(2,1)$ is 0, 6, 1, 4, 4, and $S'(2,2)$ is 0, 1, 6, 4, 4;
$S'(3,1)$ is 0, 3, 0, 3, 1, and $S'(3,2)$ is 0, 3, 0, 1, 3;
$S'(3,3)$ is 0, 0, 3, 3, 1, and $S'(3,4)$ is 0, 0, 3, 1, 3;
$S'(4,1)$ is 0, 2, 0, 1, 0, and $S'(4,2)$ is 0, 1, 0, 2, 0;
$S'(4,3)$ is 0, 2, 0, 0, 1, and $S'(4,4)$ is 0, 1, 0, 0, 2;
$S'(4,5)$ is 0, 0, 2, 1, 0, and $S'(4,6)$ is 0, 0, 1, 2, 0;
$S'(4,7)$ is 0, 0, 2, 0, 1, and $S'(4,8)$ is 0, 0, 1, 0, 2.

For each i,j we label $V(i,j)$ P_k where the kth term of $S'(i,j)$ is unity. It is not necessary to compute $S'(5,j)$ for any j, since every $V(5,j)$ can be labeled with that label which has not already appeared in the chain containing that $V(5,j)$. The result is Fig. 8.

(To be concluded.)

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A SEQUENTIAL TWO SAMPLE LIFE TEST *

BY

BENJAMIN EPSTEIN [1]

1. INTRODUCTION

Consider the following problem. A user of electron tubes is given two lots of tubes and he wishes to choose the lot having the greater mean life on the basis of a life test made on a sample of tubes drawn from each lot. To make the problem precise we assume that tubes in lots 1 and 2 have a life time which is described by the exponential[2] probability density function $\frac{1}{\theta_i} e^{-x/\theta_i}$, $x > 0$, $\theta_i > 0$, $i = 1, 2$. Generally speaking, $\theta_1 = \theta_2$, or $\theta_1 > \theta_2$, or $\theta_1 < \theta_2$. If $\theta_1 = \theta_2$, we are indifferent as to the ranking assigned to the two lots. If $\theta_1 > \theta_2$, we should prefer to have the decision procedure lead to the (correct) assertion that lot 1 has the greater mean life. Similarly if $\theta_2 > \theta_1$, we should prefer to have the decision procedure lead to the (correct) assertion that lot 2 has the greater mean life.

A measure of how lot 1 differs from lot 2 is given by the ratio $u = \max(\theta_1,\theta_2)/\min(\theta_1,\theta_2)$. In most practical problems, the experimenter would like to have a high probability of properly ranking the two lots, if the ratio of the larger mean life, $\max(\theta_1,\theta_2)$, to the smaller mean life, $\min(\theta_1,\theta_2)$, is some specified number $u_0(> 1)$. The sequential procedure given in this paper has the following properties: (i) when $u = 1$ (that is, when the mean lives θ_1 and θ_2 are equal), the probability of calling $\theta_1 > \theta_2$ (or $\theta_2 > \theta_1$) is equal to 0.5; (ii) as u increases, the probability of assigning the proper ranking to the two lots increases; (iii) when $u \geq u_0$, the preassigned ratio, then the probability of ranking the two lots incorrectly is less than or equal to some preassigned small value α.

The sequential procedure follows directly from a paper by Girshick (3).[3] Formulae for the probability of assigning a wrong ranking and for the expected number of items failed in the course of reaching a decision also follow from (3). It is interesting in this connection to note that the decision problem can be rephrased in the terminology of the problem of the ruin of the gambler (2, pp. 282–288). This is another way of finding the basic formulae of this paper.

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[2] There is considerable practical justification for this assumption. See, for example, (1). θ_i is the mean life of the tubes in lot i, $i = 1,2$.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

2. THE SEQUENTIAL PROCEDURE

The following sequential procedure attains the objectives of the first two paragraphs in the Introduction.

Let samples of size[4] n_0 be taken from each lot, and let life testing be carried out simultaneously (and continuously in time) on both lots. The sample size, n_0, is kept the same throughout the test, by replacing immediately any item which fails by a new item drawn from the same lot. Let $x_i(t)$, $i = 1, 2$, be the number of failures in sample i at time t. Let us adopt the rule that experimentation is continued so long as

$$-a < x_1(t) - x_2(t) < a, \tag{1}$$

where a is a positive integer specified in (2). As soon as $x_1(t) - x_2(t) = a$, stop experimentation and assert that $\theta_2 > \theta_1$; as soon as $x_1(t) - x_2(t) = -a$, stop experimentation and assert that $\theta_1 > \theta_2$.

The integer a is specified by imposing the additional restriction that when $u = \max(\theta_1,\theta_2)/\min(\theta_1,\theta_2) = u_0$, the probability, π_{u_0}, of making an incorrect ranking is $\leq \alpha$. If this restriction is imposed, a is the unique integer which meets the condition

$$\frac{\log \dfrac{1-\alpha}{\alpha}}{\log u_0} \leq a < \frac{\log \dfrac{1-\alpha}{\alpha}}{\log u_0} + 1. \tag{2}$$

The value of π_u, the probability of making an incorrect ranking, is given by

$$\pi_u = \frac{1}{u^a + 1}, \quad u \geq 1. \tag{3}$$

Furthermore, $E_u(n)$, the expected number of items failed in the course of reaching a decision, is given by

$$E_u(n) = a\left(\frac{u+1}{u-1}\right)(1 - 2\pi_u), \quad \text{if} \quad u > 1 \tag{4}$$

and

$$E_u(n) = a^2, \quad \text{if} \quad u = 1. \tag{5}$$

In Table I we give $E_1(n)$ and $E_{u_0}(n)$ for $\alpha = 0.01, 0.05$ and $u_0 = 3/2, 2, 5/2, 3$.

TABLE I.—*Values of* $(E_{u_0}(n), E_1(n))$ *for Various Values of* α *and* u_0.

α \ u_0	3/2	2	5/2	3
0.01	(59.1,144)	(20.7,49)	(13.9,36)	(9.92,25)
0.05	(37.0,64)	(14.1,25)	(8.88,16)	(5.57,9)

For example, if $\alpha = 0.05$, $u_0 = 3$, then $E_{u_0}(n) = 5.57$ and $E_1(n) = 9$.

[4] n_0 will not affect the number of failures required to reach a decision. However, increasing n_0 reduces the average waiting time to reach a decision. More precisely, the average waiting time to reach a decision is inversely proportional to n_0.

To obtain the sequential procedure just described, one can apply the procedures in (3) even though the present problem involves making continuous decisions in time. The particular part of (3) most relevant here is the section on double dichotomies, pp. 137–138. Our continuous life-test procedure can be considered from the double dichotomy point of view by the simple device of splitting the time axis into very small equal intervals, which we take for convenience to be our unit of measurement. In each such interval we are simultaneously observing two binomial processes which have probability of success (that is, probability of observing a failure) equal to $1/\theta_1$ and $1/\theta_2$, respectively.

Formulae (1) and (2) are obtained by writing down the appropriate sequential probability ratio at each moment t. The numerator of this ratio is the probability of observing $x_i(t)$, $i = 1, 2$, under the assumption that $\theta_1 = \min(\theta_1,\theta_2)$, while the denominator of this ratio is the probability of observing $x_i(t)$, $i = 1, 2$, under the assumption that $\theta_1 = \max(\theta_1,\theta_2)$. Defining $u = \max(\theta_1,\theta_2)/\min(\theta_1,\theta_2)$, this ratio becomes $u^{x_1(t)-x_2(t)}$. If we impose the further restriction that we want the test to lead to an incorrect ranking with probability $\leq \alpha$, if $u = u_0$, then we must have

$$\frac{\alpha}{1-\alpha} < u_0^{x_1(t)-x_2(t)} < \frac{1-\alpha}{\alpha}. \tag{6}$$

The operation of taking logarithms on (6) leads to inequalities (1) and (2), bearing in mind that $x_1(t) - x_2(t)$ can assume only integer values.

We now compute π_u, the probability of making an incorrect ranking and $E_u(n)$, the expected number of items failed in the course of reaching a decision. Using the analogy in the first paragraph, equation (1.9101) in (3) becomes (putting $\bar{a} = \bar{b} = a$), $L_u = \pi_u = \dfrac{1}{u^a + 1}$. This is our Eq. 3. Equation (1.912) in (3) has to be interpreted as the expected waiting time to reach a decision. Replacing p_1 and p_2 in (1.912) by $1/\min(\theta_1,\theta_2)$ and $1/\max(\theta_1,\theta_2)$, respectively, we get

$$E(t|\theta_1,\theta_2) = a(1 - 2\pi_u) \Big/ \left[\frac{1}{\min(\theta_1,\theta_2)} - \frac{1}{\max(\theta_1,\theta_2)} \right]. \tag{4'}$$

Let $E(n_i|\theta_1,\theta_2)$, $i = 1, 2$, denote the expected number of items in lot i, which fail in the process of reaching a decision. It can be readily shown that $E(n_i|\theta_1,\theta_2) = E(t|\theta_1,\theta_2)/\theta_i$. Then the expected number of items failed in the course of reaching a decision is given by

$$E(n|\theta_1,\theta_2) = E(t|\theta_1,\theta_2)\left[\frac{1}{\theta_1} + \frac{1}{\theta_2}\right]. \tag{4''}$$

Since $\left(\frac{1}{\theta_1}+\frac{1}{\theta_2}\right)\Big/\left[\frac{1}{\min(\theta_1,\theta_2)}-\frac{1}{\max(\theta_1,\theta_2)}\right]=\frac{u+1}{u-1}$, (4″) becomes

$$E_u(n)=a\left(\frac{u+1}{u-1}\right)(1-2\pi_u). \tag{4}$$

This completes the proof. It should be remarked that it was purely a matter of convenience to carry out the proof with $n_0 = 1$. For general n_0 only (4′) will be affected in that $E(t|\theta_1,\theta_2,n_0) = E(t|\theta_1,\theta_2,n_0 = 1)/n_0$.

Remarks

The problem of the gambler's ruin (2, pp. 282–288) is a classical example of a sequential procedure, where the terminal decisions are ruin of the gambler or of his opponent. We now indicate how π_u and $E_u(n)$ for the two sample sequential test can be found by solving an equivalent game.

The succession of Bernoulli trials which makes up the game corresponds to the succession of failures in the life test. The gambler wins (or loses) a dollar whenever the failed item is from the lot whose mean life is θ_1 (or θ_2). As a consequence of the assumption that the underlying distributions are exponential and that items which fail are replaced by new items from the same lot, it is clear that the probability of the gambler winning a dollar on a single trial is $p=\frac{1}{\theta_1}\Big/\left(\frac{1}{\theta_1}+\frac{1}{\theta_2}\right)$. Further, (1) can be written as $0<x_1(t)-x_2(t)+a<2a$. $x_1(t)-x_2(t)+a$ is the fortune of the gambler at time t. At time $t = 0$, this means that the gambler and his opponent each start with a dollars. The game goes on so long as the inequality is satisfied. It terminates as soon as either $x_1(t) - x_2(t) + a = 0$ (gambler is ruined) or $x_1(t) - x_2(t) + a = 2a$ (his opponent is ruined). Termination of the game with ruin of the gambler (opponent) is equivalent to asserting that $\theta_1 > \theta_2 (\theta_2 > \theta_1)$. Having thus set up an equivalent game we can compute the probability of ruin and expected duration of the game by means of formulae 2.4 and 3.4 (2, p. 284, p. 287). These results can be readily interpreted so as to yield (3) and (4), respectively.

3. A NUMERICAL EXAMPLE

Consider a problem where one wishes to have a probability ≤ 0.05 of assigning a wrong ranking if $\max(\theta_1,\theta_2)/\min(\theta_1,\theta_2) = 3$. In this case $\alpha = 0.05$ and $u_0 = 3$. $a = 3$ is the unique integer such that $\frac{\log 19}{\log 3} \leq a < \frac{\log 19}{\log 3} + 1$. The life test is continued so long as the inequality $-3 < x_1(t) - x_2(t) < 3$ is satisfied. The life test is terminated and a

decision is made as soon as $x_1(t) - x_2(t) = \pm 3$. π_u, the probability of assigning a wrong ranking if $\max(\theta_1,\theta_2)/\min(\theta_1,\theta_2) = u$, is given by $\pi_u = \frac{1}{u^3+1}$. If $u = 3$, $\pi_3 = \frac{1}{28}$; if $u = 2$, $\pi_2 = \frac{1}{9}$; if $u = 1$, $\pi_1 = \frac{1}{2}$. The expected number of observations required to reach a decision attains its maximum at $u = 1$. In this case $\max E_u(n) = E_1(n) = 9$. Further $E_2(n) = 7$, $E_3(n) = 5.57$, and $E_\infty(n) = 3$.

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"Snooperscope" Used to Study Silicon.—Crystals of General Electric's purest silicon, produced with great effort in the Research Laboratory, were about as transparent as a brick wall—until scientists trained a modified World War II "snooperscope" on them.

Polished silicon has a shiny metallic appearance, and looks very much like the stainless steel blade of a table knife. Using the snooperscope, Laboratory scientists have found that they can peer right through a silicon ingot several inches long, from one end to the other. In fact, it seems quite likely that this wartime device, which was first used in nocturnal observations with infrared or "dark" light, will now become a tool in industry. Working with such a device, Dr. William C. Dash, GE Laboratory staff physicist, has found he can study the interior of sections of silicon microscopically, in polarized light. In fact, he can spot contaminants, strains and crystal imperfections directly. Although visible light will not penetrate silicon, the source is nevertheless an incandescent lamp. Only the infrared or heat radiation passes through the silicon and is changed into visible light by the 'scope, thus disclosing crystal structural information.

A typical use for the 'scope is investigating strains produced when small dots of aluminum, gold, and other metals are alloyed to silicon in manufacturing transistors and rectifiers. With a snooperscope the region around a silicon alloy is seen to be highly strained, with a deeply penetrating "rising sun" pattern, resembling a window struck by an air rifle pellet. The novel instrument has also disclosed the presence of markings in silicon ingots which arise from fluctuations in growth rate. Rendered visible by the industrial snooperscope, such markings and many other phenomena are studied for their possible effects upon the quality of commercial silicon semiconductor devices.

Sewage Sterilization by Gamma-Radiation.—The deadly radiation from radioactive substances, already serving mankind in industry, agriculture and medicine, may be put to other beneficial uses according to investigations conducted by the Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Va., in conjunction with the Oak Ridge National Laboratory at Oak Ridge, Tennessee.

After six months of research, sanitary engineers have demonstrated that water-borne sewage can be sterilized by gamma-radiation without activating or leaving residual radiation in the treated liquids.

The waste products from atomic power plants offer one possible source of the required radioactive materials. According to Mr. Harry N. Lowe, Chief of the Engineer Laboratories' Sanitary Engineering Branch, the possibility exists that these waste products can be used to sterilize water, sewage and other liquids. Work is continuing to fully exploit other possible benefits.

Besides the possible use of gamma-radiation by the Corps of Engineers for the sterilization of sewage other agencies have found that gamma rays can also be used to sterilize certain drugs and antibiotics which are harmfully affected by heat, to preserve meat and other food stuffs by killing the bacteria that cause spoilage, to irradiate pork and thus inhibit the spread of trichinosis and to decrease the number of bacteria in swimming pools without using chlorine, etc.

SECONDARY ELECTRON EMISSION BY PRIMARY ELECTRONS IN THE ENERGY RANGE OF 20 KILOVOLTS TO 1.3 MEV *

BY

BERNARD L. MILLER [1] AND W. C. PORTER [2]

INTRODUCTION

About the summer of 1951 at The Bartol Research Foundation, the operation of a small linear electron accelerator (1),[3] which can be used flexibly to provide electron beams in the region 20 kev to 1.3 Mev, became sufficiently well-defined to shift the interest from its performance to its application in research. An experiment on secondary emission caused by high energy electron bombardment was considered a suitable subject for a first investigation with this accelerator, since it could be compared with the work of Trump and Van de Graaff (2) in the range 30 to 350 kilovolts and then the energy of the primary electrons extended to the 1.3 Mev limit of the linear accelerator. The electron beam of the linear accelerator consists of a high energy bunch comprising about fifty per cent of the output with the remainder spread over lower energies. Accordingly, the beam was passed through a magnetic analyzer having a resolution ± 4 per cent in momentum before entering the secondary emission assembly. This spectrometer was calibrated by obtaining absorption curves of the analyzed beam in aluminum; the ten best curves, taken at various energies, were used to obtain a mean value for the spectrometer constant.

EXPERIMENTAL SET-UP

The secondary emission assembly is shown in Fig. 1. The target holder and secondary electron collector are insulated from each other by glass sections which slip over "O" rings for vacuum seals. The vacuum was of the order of 10^{-5} mm Hg. The targets were $\frac{3}{4}$-in. diameter, usually $\frac{1}{8}$ in. thick. The secondary collector was made of a low secondary emission material, aluminum, to reduce the effect of secondary emission from the collector itself.

The resolved beam current was of the order of 0.1 milliampere during 1 microsecond pulses, usually repeated 100 times per second. The target and collector currents were each passed to ground through an averaging resistor-condenser (10^9 ohms, 100 $\mu\mu$f) parallel combina-

* Assisted in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

tion across which the voltage was read with Cenco electronic voltmeters. The impedance elements were covered with paraffin wax to reduce effects caused by surface leakage and their relative resistance was checked on each run; also, the relative meter sensitivities were checked by measuring the same battery voltage. A battery of reversible polarity in the target lead allowed a voltage difference up to 400 volts between collector and target. It was necessary to use a grounded cage

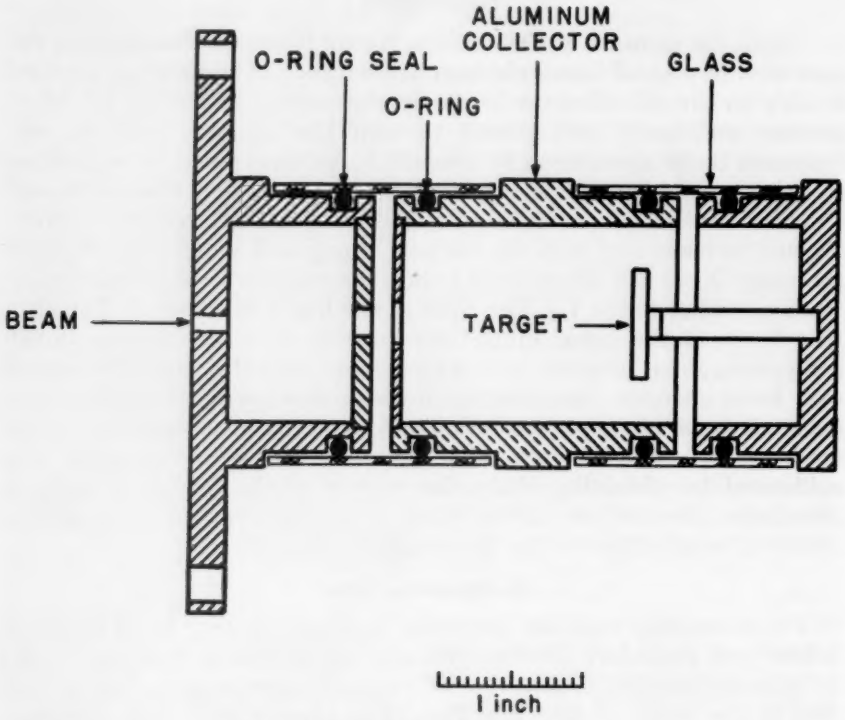

FIG. 1. Secondary emission assembly.

fitting closely around the target assembly and a coaxial cable to the meter to eliminate spurious currents due to the ionization of the surrounding air by X-rays generated at the target.

To check the undesirable possibility that some of the high energy primary electrons might scatter from the defining slits directly to the secondary collector, a 6-in. tube lined and terminated with graphite (a poor secondary emitter) was used as a target to catch the primary beam; with a small negative voltage on the collector to hold back any slow secondaries from the tube target, no current was observed to the collector, showing that the total primary beam went to the target.

RESULTS

The data obtained were in qualitative agreement with those of Trump and Van de Graaff in the lower energy range. The secondaries consisted of two groups, one of low energy (mainly less than 30 volts) and a fast component of possibly elastically scattered primaries. This feature is

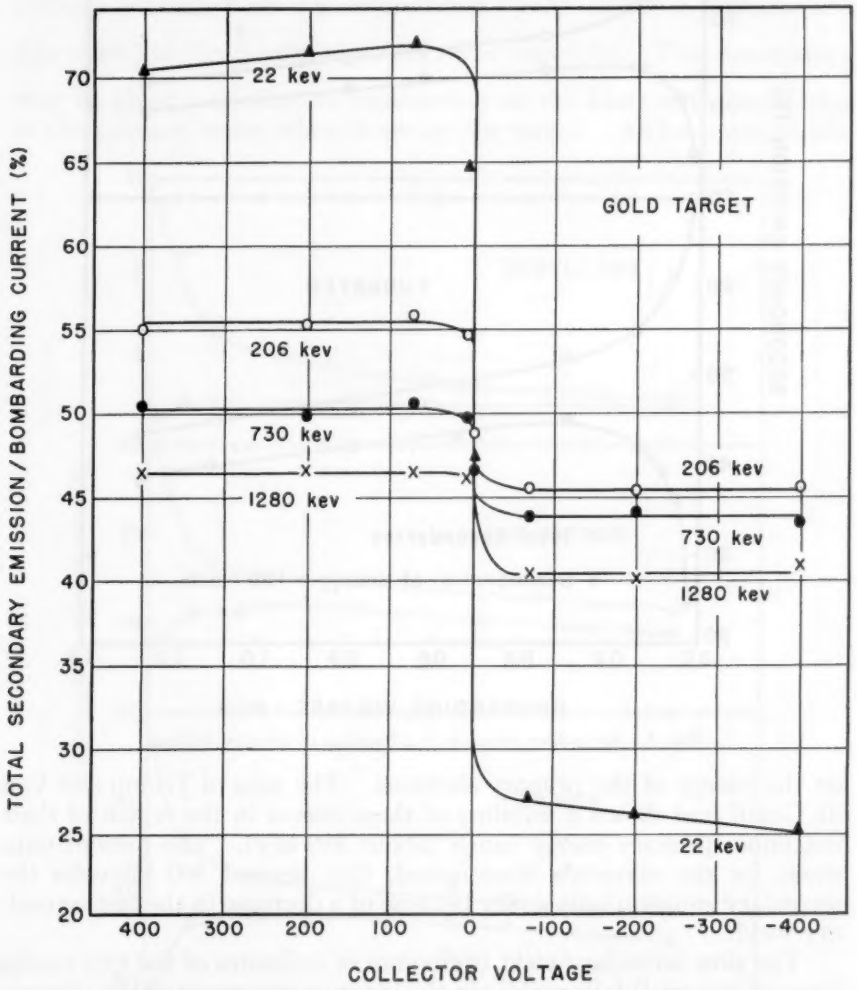

FIG. 2. Secondary emission as a function of collector voltage.

illustrated in Fig. 2 showing the secondary emission dependence on collector potential for different primary energies. From such data, it is natural to plot two curves for each target material as in Figs. 3 and 4, showing the dependence of the two components of secondary emission

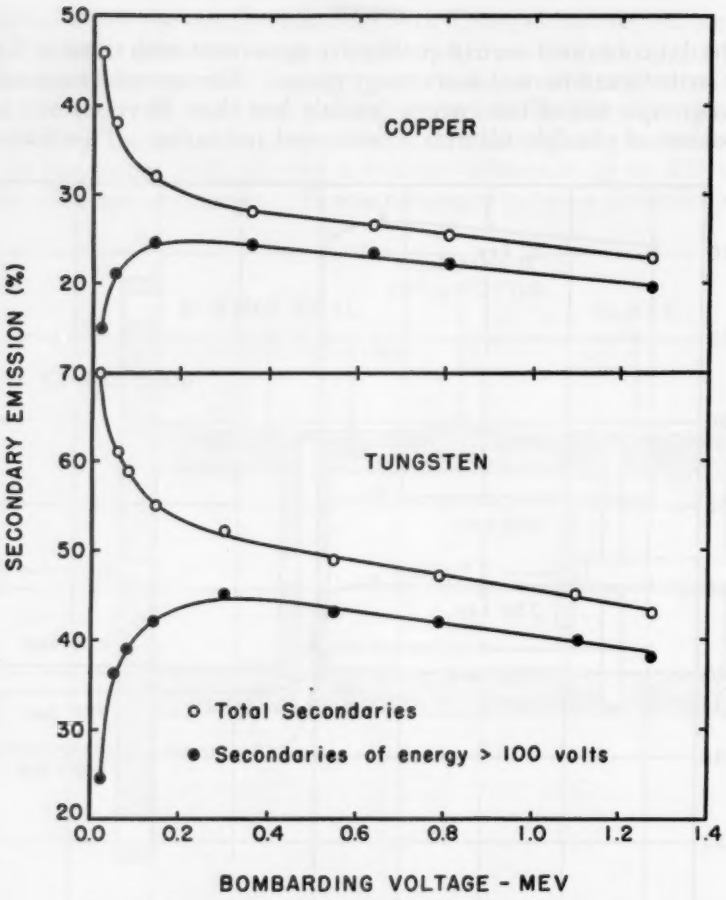

FIG. 3. Secondary emission as a function of primary voltage.

on the energy of the primary electrons. The data of Trump and Van de Graaff had shown a levelling of these curves in the region of their maximum primary energy range (about 350 kev). The present data show, for the materials investigated, that beyond 300 kilovolts the secondary emission falls slowly because of a decrease in the fast secondary yield.

The slow secondary yield (difference of ordinates of the two curves for each material) falls rapidly in the lower energy range as the primary voltage increases and then becomes nearly constant in the upper half of the energy range covered. This is quite similar to the way the intensity of ionization theoretically depends on the primary energy. A formula for rate of energy loss by electrons due to ionization (3) gives the ionization as proportional to $\frac{1}{v^2} f(v)$, where v is the electron velocity and

$f(v)$ varies slowly. The $\frac{1}{v^2}$ term dominates in the non-relativistic region, whereas the slowly increasing second term causes a minimum ionization in the 1 to 2 Mev range. Calculations for tungsten and copper, and comparison with the data, showed that the observed slow secondary emission decreased faster than predicted by the ionization formula, at a rate more like the $\frac{1}{v^2}$ term alone for v less than 0.9 c. This discrepancy may be given a qualitative explanation on the basis of the scattering of the primary beam when it enters the target. At low energies the

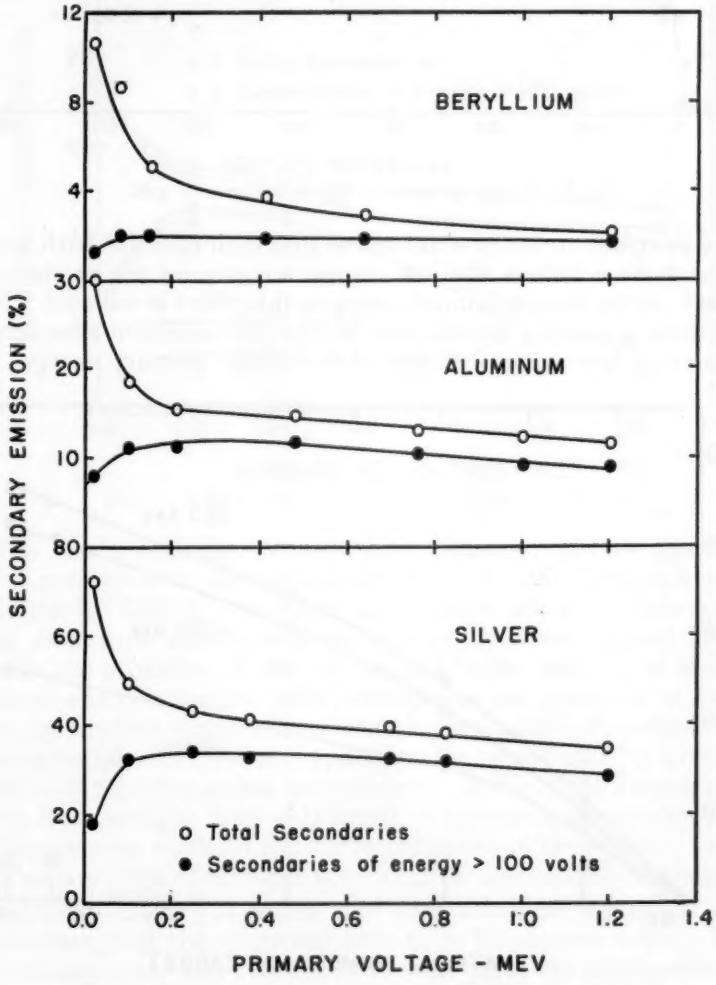

FIG. 4. Secondary emission as a function of primary voltage.

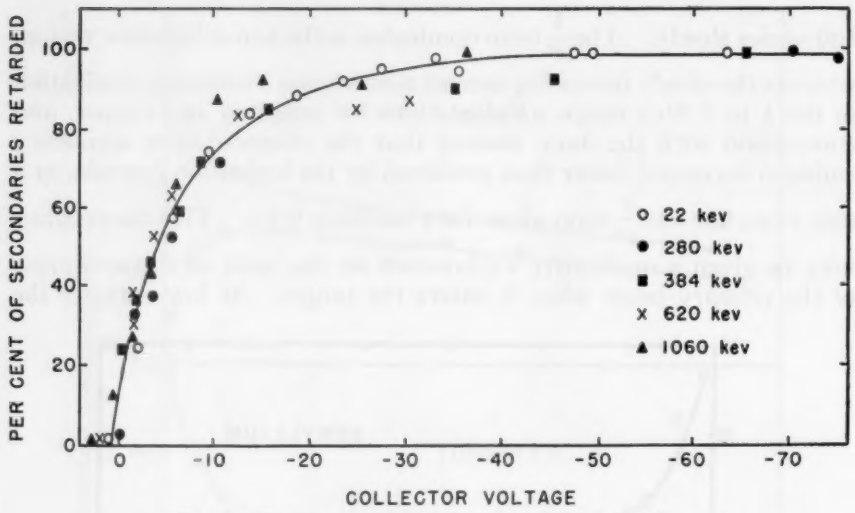

FIG. 5. Energy spectrum of slow secondaries for gold.

primary electrons are easily scattered so that their effective path lengths near the surface (where the low energy secondaries are produced) is increased; at the higher primary energies this effect is reduced. Relatively, then, scattering should increase the production of slow secondaries more at low primary energy than at high primary energy, thus

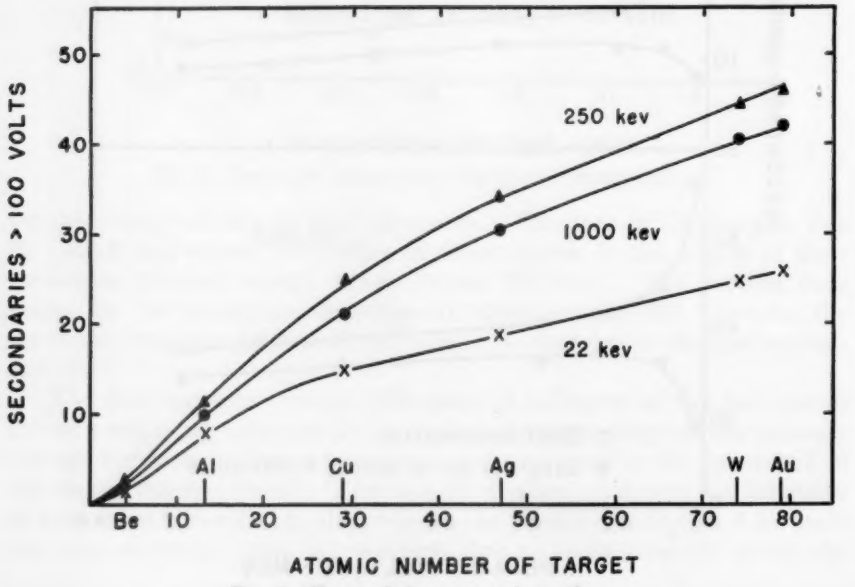

FIG. 6. Fast emission *vs.* atomic number.

accounting for the more rapid reduction in slow secondaries as the primary energy is increased.

The energy spectrum of the slow secondaries was measured for a gold target at different primary energies. In Fig. 5 is shown the fractional decrease in the slow secondary yield as the collector voltage is made more negative, for a number of different primary energies. Although there is appreciable scatter of points, the data indicate that the energy distribution of the slow secondaries was rather independent of the primary voltage.

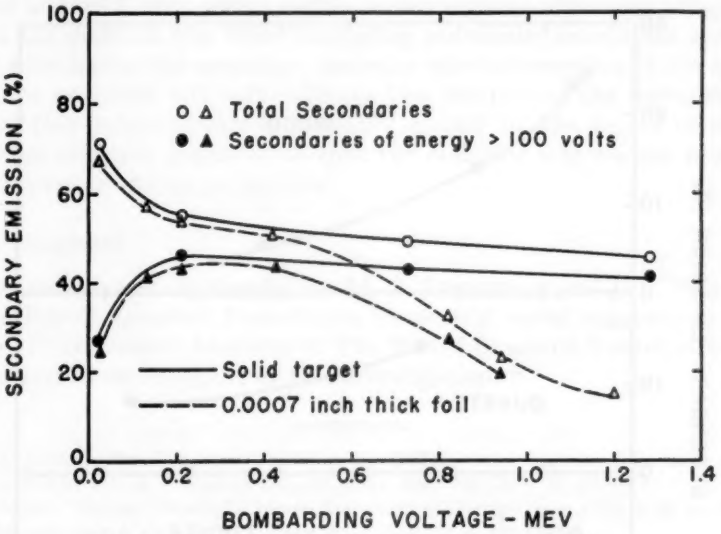

FIG. 7. Solid and thin foil gold target secondary emissions.

The fast component of secondary emission, plotted against the atomic number of the targets is shown in Fig. 6. Similar data have been published by Saldick and Allen (4, 5); there are some discrepancies, which aside from experimental error, may have been caused by differences in the geometry of the set ups and in the material of which the collector was constructed (slow secondaries are produced at the collector by the fast target secondaries, in an amount dependent on collector material, and these are repelled to the target, causing a reduction in the value recorded as fast secondaries). In Fig. 6 the drooping of the curves from straight lines is believed to represent the shielding effect of inner electron shells on the scattering power of the nuclei. The root mean square angle of multiple scattering (6) theoretically depends on Z as $\sim Z\sqrt{\ln[E/E_0 Z^{4/3}]}$ in which $E = \frac{1}{2}mv^2$ (classical expression for the kinetic energy of the electrons) and $E_0 = 27$ electron volts. This Z dependence agreed well with the 22 kilovolt curve but only qualitatively with the 1 Mev curve.

In Fig. 7 is shown a comparison between a thick gold target and a thin foil of gold. The slow emission is about the same for the two targets, there being no point in a close comparison because of possible differences in the state of the two surfaces. The fast emission, however, if due to scattered primaries, should be affected when the range of the primaries exceeds twice the foil thickness. The 0.7-mil foil had a mass of 34 milligrams/cm²; to penetrate the foil to the far side and return, an electron would require an energy corresponding to a 68-mg/cm² range, which means about a 275-kv electron. Thus only for primary

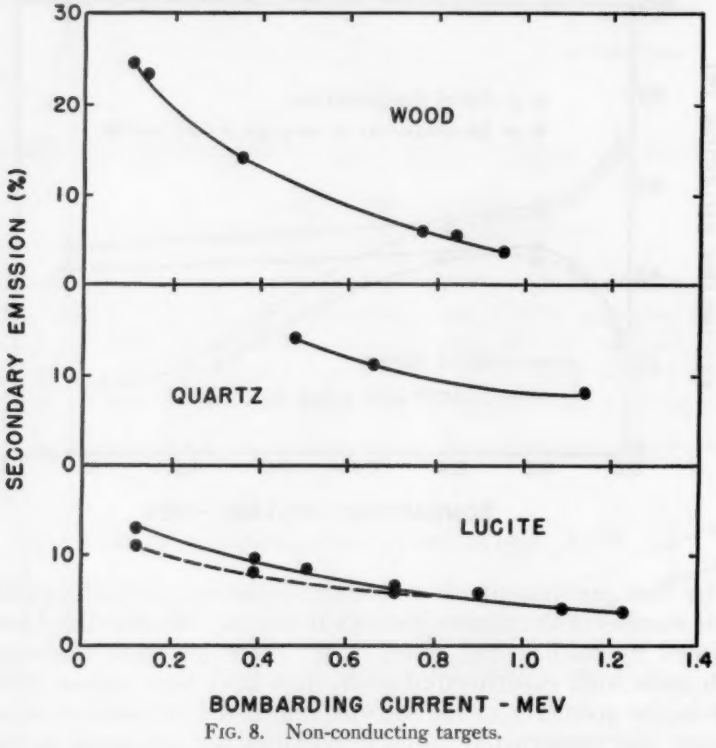

FIG. 8. Non-conducting targets.

energies exceeding 275 kilovolts could electrons make use of scattering material at a depth beyond the foil thickness as far as contributing to the back scattered emission is concerned. Experimentally a decline in the number of fast secondaries from the foil, as compared with the thick target, set in at about 400 kilovolts.

Finally, some experiments were undertaken with insulators as targets. These experiments were preliminary in nature, with three targets selected to roughly represent a poor insulator, a good insulator, and an excellent insulator: namely, wood, lucite, and quartz. The target thickness was chosen at half the range of the beam at 1 Mev in

the target material. At 22-kv primary energy the secondary emission ratio rose to near unity as apparently the target charged to a potential such that no primary electron could reach it. In the region 100 to 200 kilovolts intermittent violent discharges of the target occurred after each of which the secondary emission fell to a minimum and slowly increased until the next discharge; the secondary emission value immediately after a discharge was recorded as typical of the uncharged material. In the region 300 to 400 kv the charging and discharging became only a flickering of the meter needles which practically disappeared around 1 Mev where stable emissions were obtained. Figure 8 shows the data on the three insulating materials; except for a small effect with lucite, the secondary emission was independent of the available plus or minus 400 volt collector bias relative to the metal target holder; this independence is probably caused by the target charging to a high negative potential so that the collector was always positive with respect to the target surface.

Acknowledgment

The writers wish to thank Drs. M. A. Pomerantz and J. F. Marshall of The Bartol Research Foundation for several useful suggestions, and Dr. W. F. G. Swann, Director of The Bartol Research Foundation, for his interest in and support of this investigation.

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New 10-in. Diffusion Pump.—A new versatile high vacuum diffusion pump for moderate sized vacuum systems has been announced by the Equipment Division of National Research Corporation. The Model H-10-P pump is a 10-in. diameter fractionating diffusion pump which can also be used as a booster pump. When using Narcoil-40 pump fluid, the pump has a speed of 1700 liters per second in the pressure range from 5×10^{-4} to 5×10^{-5} millimeter of mercury and has a throughput of 3800 liters per second at 10 microns. It blanks-off below 1×10^{-6} mm. Hg and has a maximum tolerable forepressure of 200 microns.

The H-10-P pump operates as a booster pump when Narcoil-10 is used as the pumping fluid. Pumping speed at 1 micron is 2000 liters per second and the pump has a throughput of 6000 liters per second at 10 microns. Blank-off is better than 1.5×10^{-4} mm. Hg and maximum tolerable forepressure is 400 microns.

New High Temperature Steatite-Cased "Miniroc" Capacitors.—Cornell-Dubilier Electric Corp., of South Plainfield, N. J., has announced a new series of tubular sub-miniature capacitors that require no voltage derating even at + 130° C., and which provide maximum moisture resistance actually exceeding MIL-C-91 requirements. These improved characteristics are made possible through advanced C-D processing techniques and the use of steatite-tubular cases and Myler polyester dielectric. Exceptionally high insulation resistance is achieved throughout their entire temperature range.

To insure low RF impedance, the non-inductively wound extended foils are firmly soldered to the wire leads. Great care is taken to assure a low resistance connection throughout.

The capacitor ends are sealed with C-D's famous Polykane fill which tightly bonds to the steatite walls and to the lead wires to prevent the entrance of moisture and thus enable the units to withstand extremes of handling, soldering temperature and vibration.

Scintilogger Measures Radioactivity to 6000 ft.—The new Deep Drill Hole Scintilogger announced by The Radiac Company, Inc., New York City, is designed for radioactivity logging of drill holes to 6000 ft. Its verified in-hole-out-hole radiation profiles indicate deeply buried uranium deposits and identify oil productive zones in new wells as well as new productive zones in old wells.

The Scintilogger is a portable, self-contained well logging system which includes a scintillation well logging probe, a minimum 2000 ft. of cable, surface control box and graphic strip chart recorder synchronized to a footage counter (mounted on a stand containing the playout and take-up reel). The stainless steel probe, which is 36 in. long and 2 in. in diameter, may be weighted to assure complete descent through water and mud in uncased holes. It is mounted on a steel cable having a breaking strength of 1500 lb. and a rated strength of 900 lb.

Time constants (1, 4, and 16 sec.) and sensitivity ranges (600, 1800 and 6000 cpm.) are controlled at the surface to provide the operator with as much or as little detail in his log as he desires. Sections of interest can be re-logged at expanded scale or at other sensitivities while in the well. The log can be correlated with electric logs and cores while in operation and graphically illustrates changes in lithology exactly as they occur, with the same sharp breaks or gradations.

Because of the Scintilogger's accurate correlation with core samples, it is used to supplant expensive core drilling operations.

ON THE SPECIFICATION OF BÔCHER EQUATIONS

BY

PARRY MOON [1] AND DOMINA EBERLE SPENCER [2]

INTRODUCTION

Klein (1)[3] and Bôcher (2) suggested that many of the ordinary differential equations obtained in mathematical physics by separation of variables can be derived from certain prototype equations. The equations considered are linear and of the second order:

$$\frac{d^2Z}{dz^2} + P(z)\frac{dZ}{dz} + Q(z)Z = 0, \tag{1}$$

with P and Q rational functions of z. The Bôcher prototypes are frequently taken with all singularities of the first order and with exponents 0 and $\frac{1}{2}$ at each singularity. Confluent equations are then obtained from the prototypes by allowing some of the simple poles to coalesce.

The Bôcher equations provide a convenient foundation for building a classification of the ordinary differential equations of mathematical physics. Such a classification would be valuable in indicating relations among various equations and in showing when two apparently distinct equations are actually equivalent and have the same form of solution. Evidently a satisfactory specification must be unambiguous: it must provide

(*a*) Different specifications for all equations whose solutions involve different functions,
(*b*) The same specification for all equations whose solutions are the same.

A classification of equations, based on Bôcher's work, is given by Ince (3). Unfortunately, his method does not satisfy either of the above requirements. A modified method is described in the following pages, and this new specification is believed to give unambiguous designations.

SPECIFICATION

Consider second-order equations of the form of Eq. 1. We now introduce the restriction that

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[2] Department of Mathematics, University of Connecticut, Storrs, Conn.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

$$\left.\begin{aligned} P(z) &= \frac{1}{2}\left[\frac{m_1}{z-a_1}+\frac{m_2}{z-a_2}+\cdots+\frac{m_{n-1}}{z-a_{n-1}}\right], \\ Q(z) &= \frac{A_0+A_1z+A_2z^2+\cdots+A_lz^l}{(z-a_1)^{m_1}(z-a_2)^{m_2}\cdots(z-a_{n-1})^{m_{n-1}}}. \end{aligned}\right\} \tag{2}$$

Here n = number of distinct singular points, including one at infinity; m_i, l = integers. Poles occur at $z = a_1, a_2, \cdots a_{n-1}, \infty$. The order of the pole at ∞ is, of course, equal to the greater of the two integers designating the orders of $(2z - z^2P(z))$ and $z^4Q(z)$ as $z \to \infty$. The total order of singularities is $h = \sum_{i=1}^{n} m_i$.

We shall call the equations defined by (1) and (2) *Bôcher equations*. These equations are essentially those considered by Bôcher; but we obtain them directly from Eq. 2 instead of by confluence from a prototype. This modification tends to simplify the subject and leads to a more compact classification than is obtained by Bôcher and Ince (3).

The new *specification* is merely a sequence of integers representing the orders of the poles: $\{m_1, m_2, \cdots m_{n-1}, m_n\}$, where m_n is the order of the pole at ∞. For instance, the Legendre equation may be written

$$\frac{d^2Z}{dz^2}+\frac{1}{2}\left[\frac{1}{z-a_1}+\frac{2}{z-a_2}\right]\frac{dZ}{dz}+\left[\frac{A_0+A_1z}{(z-a_1)(z-a_2)^2}\right]Z=0.$$

Evidently this is a Bôcher equation with $n = 3$, $m_1 = 1$, $m_2 = 2$, $m_3 = 0$, $l = 1$. The equation has a simple pole at $z = a_1$ and second-order poles at a_2 and ∞. The specification is therefore $\{122\}$. The final integer m_n in our specification holds a privileged position and must not be moved; the other integers may be commuted at will.

CLASSIFICATION

The foregoing method of specification leads to a convenient way of *classifying* the ordinary differential equations of field theory. The basic division is with respect to the number n of distinct singular points. In the class $n = 4$, for example, we have $\{2222\}$, $\{1222\}$, $\{1122\}$, $\{1112\}$ as the only possible equations with regular singularities and with a second-order pole at ∞. Each of these equations has a degenerate form obtained by setting $A_l = 0$: $\{2221\}$, $\{1221\}$, $\{1122\}$, $\{1111\}$. If the point at ∞ is an irregular singularity, various degenerate equations can be written with $A_l = 0$, $A_l = A_{l-1} = 0$, . . . But not all the possible equations actually occur in field theory.

A classification that includes all the separation equations of field theory, for 40 coordinate systems (4), is given in Table I. Extension of the table to $n > 4$ and to higher-order poles is obvious, but such extension is not needed in physics or engineering.

TABLE I.—*Classification of Bôcher Equations*

n	Name	Original	Degenerate Cases
4	Cyclidal equation II	{1222}	
	Cyclidal equation I	{1122}	{1121}
	Lamé wave equation	{1113}	{1112}, {1111}
3	Legendre wave equation	{123}	{122}, {121}
	Paraboloidal wave equation	{114}	{113}
2	Bessel wave equation	{24}	{23}, {22}
	Weber equation	{14}	
	Elementary	{33}	{32}
1	Elementary	{04}	{01}

TRANSFORMATIONS

Many of the ordinary differential equations occurring in physics are not of the Bôcher form but can be put into this form by a transformation of the independent variable. Two differential equations may be said to be *equivalent* if one can be obtained from the other by an elementary functional transformation.

The question then arises as to whether the uniqueness of our equation specification is endangered by this flexibility introduced by transformation of variable. In other words, given a Bôcher equation, under what circumstances does an elementary transformation give another Bôcher equation having a different designation?

Consider the Bôcher equation,

$$\frac{d^2Z}{d\zeta^2} + \frac{1}{2}\left[\frac{m_1}{\zeta} + \frac{m_2}{\zeta - a_2} + \cdots + \frac{m_{n-1}}{\zeta - a_{n-1}}\right]\frac{dZ}{d\zeta} + Q(\zeta)Z = 0. \quad (3)$$

Here a_1 has been set equal to zero, which can be done without loss of generality. The transformation $z = \zeta^p$ gives

$$\frac{d^2Z}{dz^2} + \frac{1}{2}\left[\frac{(2p+m_1-2)/p}{z} + \frac{1}{pz^{(p-1)/p}}\left(\frac{m_2}{z^{1/p}-a_2} + \cdots + \frac{m_{n-1}}{z^{1/p}-a_{n-1}}\right)\right]\frac{dZ}{dz} + \frac{1}{p^2z^{(2p-2)/p}}Q(z)Z = 0. \quad (3a)$$

The first term within the brackets is of Bôcher form but the others are not, except for the trivial case of $p = 1$. Therefore, *a power-function transformation of a Bôcher equation cannot possibly yield a Bôcher equation if the equation has more than two singularities.*

For an equation with two singularities, Eq. 3*a* reduces to

$$\frac{d^2Z}{dz^2} + \frac{1}{2}\left[\frac{(2p + m_1 - 2)/p}{z}\right]\frac{dZ}{dz} + \frac{1}{p^2}\left[\frac{A_0 + A_1 z^{1/p} + \cdots + A_l z^{l/p}}{z^{(2p+m_1-2)/p}}\right]Z = 0. \quad (3b)$$

The exponent in the denominator of Q is equal to the numerator of P, as required for a Bôcher equation; but the numerator of Q is of Bôcher form only under very unusual conditions. Similar conclusions are reached for the other elementary functional transformations. We conclude, therefore, that in general a Bôcher equation does not transform into a Bôcher equation, and thus the process of transformation cannot introduce ambiguity into our specification except with the simplest of equations.

There remains the consideration of these simple equations, where transformation of a Bôcher equation may yield a Bôcher equation. The results are assembled in Table II, which indicates that these 11

TABLE II.—*Simple Bôcher Equations.*

Transformation	Specification	Equation
—	{04}	$\frac{d^2Z}{dz^2} + A_0 Z = 0.$
$\zeta = z^2$	{13}	$\frac{d^2Z}{d\zeta^2} + \frac{1}{2\zeta}\frac{dZ}{d\zeta} + \frac{A_0}{4\zeta}Z = 0.$
$\zeta = 1/z$	{40}	$\frac{d^2Z}{d\zeta^2} + \frac{2}{\zeta}\frac{dZ}{d\zeta} + \frac{A_0}{\zeta^4}Z = 0.$
$\zeta = 1/z^2$	{31}	$\frac{d^2Z}{d\zeta^2} + \frac{3}{2\zeta}\frac{dZ}{d\zeta} + \frac{A_0}{4\zeta^3}Z = 0.$
$\zeta = e^z$	{22}	$\frac{d^2Z}{d\zeta^2} + \frac{1}{\zeta}\frac{dZ}{d\zeta} + \frac{A_0}{\zeta^2}Z = 0.$
$\zeta = \cosh^2 z$	{112}	$\frac{d^2Z}{d\zeta^2} + \frac{1}{2}\left[\frac{1}{\zeta} + \frac{1}{\zeta - 1}\right]\frac{dZ}{d\zeta} + \frac{A_0}{4\zeta(\zeta - 1)}Z = 0.$
—	{01}	$\frac{d^2Z}{dz^2} = 0.$
$\zeta = 1/z$	{10}	$\frac{d^2Z}{d\zeta^2} + \frac{2}{\zeta}\frac{dZ}{d\zeta} = 0.$
$\zeta = z^p$ $(p \neq -1)$	{11}	$\frac{d^2Z}{d\zeta^2} + \frac{1}{2}\frac{2(p-1)/p}{\zeta}\frac{dZ}{d\zeta} = 0.$
$\zeta = e^{az}$	{11}	$\frac{d^2Z}{d\zeta^2} + \frac{1}{\zeta}\frac{dZ}{d\zeta} = 0.$
$\zeta = \cosh^2 z$	{111}	$\frac{d^2Z}{d\zeta^2} + \frac{1}{2}\left[\frac{1}{\zeta} + \frac{1}{\zeta - 1}\right]\frac{dZ}{d\zeta} = 0.$

equations occurring in field theory are all equivalent to either {04} or {01}. We stipulate that in cases where several Bôcher equations are equivalent, the simplest specification (lowest n, lowest h, highest-order singularity at ∞) is employed. This accounts for a number of gaps in Table I, where equivalent equations have been omitted.

THE INCE SPECIFICATION

Now consider how the new specification differs from that of Ince (3). The latter designates an equation by $[a,b,c]$, where a = number of simple poles, b = number of second-order poles, c = number of poles of order greater than 2. For a third-order pole, $c = 1$; for a fourth-order pole, $c = 1_2$, etc.

To show that the Ince method of specification is ambiguous, we need merely introduce two examples, one violating (a) and the other (b) of the requirements mentioned in the introduction of this paper. For (a), take the equations

$$\left.\begin{aligned} \frac{d^2Z}{dz^2} + \frac{1}{z}\frac{dZ}{dz} + \frac{A_1 z}{z^2} Z = 0, \\ \frac{d^2Z}{dz^2} + \frac{3}{2z}\frac{dZ}{dz} + \frac{A_1 z}{z^3} Z = 0. \end{aligned}\right\}$$

Both are Bôcher equations. The first equation is specified by {23} and its solution is in terms of Bessel functions. The second is specified by {32} and its solution is in terms of power functions. The two equations are distinct and should have different specifications. Yet the Ince specification is $[0,1,1]$ in each case.

For (b), take

$$\left.\begin{aligned} \frac{d^2Z}{d\zeta^2} + \frac{1}{\zeta}\frac{dZ}{d\zeta} + \left(\frac{-p^2 + q^2\zeta}{\zeta^2}\right) Z = 0, \\ \frac{d^2Z}{dz^2} + \frac{1}{z}\frac{dZ}{dz} + \frac{1}{4}\left(\frac{-p^2 + q^2 z}{z^2}\right) Z = 0. \end{aligned}\right\}$$

The first is Bessel's equation, the second is the same equation with the independent variable transformed according to $z = \zeta^2$. According to our rule of specifying only the equation with smallest h, we take the second form and list it as {23}. Ince has no such rule, so he lists the first equation as $[0,1,1_2]$ and the second as $[0,1,1]$. In classifying equations by the Ince method, therefore, the same equation may appear with several different specifications.

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NUCLEAR ENERGY NEWS

ENGINEERING TEST REACTOR

The Atomic Energy Commission has invited industry to build and operate with private funds an engineering test reactor which would provide certain radiation facilities the Commission will need early in 1957 for the development of cores, fuel assemblies, and other components of reactor projects of the Air Force, Navy, Army, and civilian nuclear power program. The AEC will contract with the owner and operator of such a reactor for use of a substantial amount of the irradiation capacity of the reactor over a five-year period.

Irradiation facilities now used by the AEC are not suitable for engineering experiments which require relatively large spaces in areas with high neutron intensities. The largest suitable irradiation space now available is one hole about five inches square at the Materials Testing Reactor. The AEC's requirements include several larger spaces extending through the reactor, completely surrounded by fuel, and providing very high neutron intensities.

The AEC has invited industry to submit by July 1, 1955, proposals to finance, construct, and operate an engineering test reactor (ETR) which will meet the new requirements. It is to include the following characteristics:

(1). One test hole $9'' \times 9''$ extending through the reactor, and completely surrounded by fuel.
(2). One test hole $6'' \times 9''$ with the same characteristics as (1).
(3). Three test holes $6'' \times 6''$, one of which can provide a $6'' \times 9''$ test space, with the same characteristics as (1).
(4). Two test holes $3'' \times 3''$ in the active lattice portion of the core surrounded by fuel on all four sides and extending through the reactor.
(5). Four test holes $3'' \times 3''$ in the active lattice or reflector portion of the core, not necessarily surrounded by fuel on all four sides. It will not be necessary that all of these holes extend completely through the reactor.
(6). The ability to provide in these test holes, over a length of $24''$ to $36''$, uniform average thermal neutron fluxes of 4×10^{14} $n/cm^2/sec$ and uniform average above—thermal fluxes greater than 10^{15} $n/cm^2/sec$ as measured in the midplane of the reactor core.
(7). Flexibility in the core design to make possible the insertion of specimens as large as $15''$ in diameter.

The Phillips Petroleum Company, operator of the Materials Testing Reactor at the National Reactor Testing Station (NRTS) near Idaho Falls, Idaho, has prepared a conceptual design of an engineering test reactor which has the above general characteristics. This conceptual design, which need not be utilized, is available. Also, certain information relative to the design, construction, and operation of the Materials Testing Reactor will be made available.

In addition to contracting with the owner and operator of the proposed reactor for a substantial amount of the irradiation capacity of the reactor, the AEC will: (1) consider furnishing the required special nuclear material free of use-charge for the first five years of operation, and at Commission-approved charges thereafter; (2) consider leasing space for the reactor at the National Reactor Testing Station (NRTS) in Idaho as an optional location; (3) process spent fuel elements at charges established by the Commission; and (4) enter into access agreements covering the furnishing of ETR design drawings and specifications, as available, to organizations interested in submitting proposals and desiring such information.

The AEC estimates that it would cost between $10 million and $15 million to build the engineering test reactor at the NRTS, where certain existing services could be utilized. The AEC desires that construction of the ETR start no later than October 1955 and be completed within 15 months, so that operation at design characteristics may begin not later than March 1957.

IMPROVED DESIGN FOR REACTOR AND HEAT EXCHANGER

A new design of nuclear reactor has been proposed by Dr. E. A. Luebke, physicist, and L. B. Vandenberg, engineer, of the Knolls Atomic Power Laboratory in Schenectady, N. Y., operated by the General Electric Company for the Atomic Energy Commission. It is intended to result in a considerable decrease in the weight per horsepower of the reactor, which would have an important bearing on the application of a nuclear power plant to aircraft, locomotives, and ships.

Much of the weight and bulk of the conventional reactor has been removed in the new design by "nesting" the reactor inside a cylindrical heat exchanger. This arrangement helps to confine radioactivity to the reactor and therefore requires less shielding.

Another feature of the design is a self-pumping action for circulating liquid sodium in the heat exchanger. This pumping action will result from an electric current flowing between the pole pieces of a cylindrical horseshoe magnet mounted in the heat exchanger. The electric current will be generated by a thermocouple effect resulting from the temperature gradient normally existing between the hot and cold tubes of the heat exchanger. The pumping action will be largely self-regulating, but in addition will have an electrical control. This means

of circulating the liquid metal coolant eliminates much of the piping and valves ordinarilly used and also gets away from some of the problems involved in the use of conventional pumps.

ADDITIONAL POWER REACTOR STUDY GROUPS

The Atomic Energy Commission has announced approval of two additional nuclear power reactor groups. (Earlier ones were listed in the June JOURNAL, p. 531.) The purpose of these studies is to determine the nuclear power system or systems most practical, technically and economically, for future construction and operation in the areas served by the member utilities and to estimate when such construction and operation will prove economically feasible.

The one group is the Puget Sound Utilities Council composed of five utility organizations in the State of Washington. The other group is the Seminole Electric Cooperative, Inc., a federation of five electric distribution cooperatives in north-central Florida. This is one of the highest fuel cost areas in the country and therefore a likely place where a power reactor might be economically feasible.

ACCESS TO NUCLEAR TECHNOLOGY INFORMATION

The Atomic Energy Commission announced on April 20 a new program under which organizations or individuals may be given access to non-military confidential and secret restricted data on atomic energy technology for their own private purposes. The program simplifies and replaces the provisional industrial participation program which made classified restricted data in the field of reactor technology available under three distinct types of agreements.

Under the new program, non-military information which is classified as "confidential, restricted data," may be made available to any person who can evidence a potential use or application of the information in his business, profession, or trade. The other conditions to this access are that the applicant obtain a simplified security ("L") clearance and agree in writing to conform to all AEC security regulations. The Government will waive all rights in inventions and discoveries arising out of access to such information. Where an invention or discovery is made or conceived as the result of access under this program the applicant will waive potential claims against the Government arising from the imposition of secrecy orders on patent applications and for awards under the Atomic Energy Act of 1954.

Under similar conditions, limited access may also be granted to certain specific information, classified as "secret, restricted data," if the applicant can demonstrate that such information has an immediate and significant effect on his business, profession, or trade. However, in this case the applicant must obtain a full security ("Q") clearance

and the Government will retain royalty-free non-exclusive rights for governmental purposes in inventions and discoveries which result from such access.

Present AEC contractors will be granted access to both categories of restricted data for private purposes on the same basis and conditions and will be granted the same rights as other applicants.

All persons or groups given access to restricted data will be required to reimburse the Government for readily identifiable costs of services rendered in connection with such access, including the cost of security clearances in excess of a certain minimum number and the cost of publications or documents provided.

So-called "study agreements" now in effect will be converted to the new type arrangements.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS

INSULATING SEAL FOR HIGH-PRESSURE EQUIPMENT

In connection with work on high-pressure standards, the National Bureau of Standards has devised a special insulating seal which effectively solves the problem of leakage around electrical connections to high-pressure vessels. Simply constructed of inexpensive materials, the high-pressure seal utilizes a sapphire bushing to obtain the necessary combination of high mechanical strength and good electrical insulating properties. The device has successfully withstood pressure up to 170,000 pounds per square inch. It was designed by H. A. Bowman and associates of the Bureau staff working under the sponsorship of the Army Ordance Corps.

Methods of measuring and controlling high pressures are becoming increasingly important to science and industry. For example, in the chemical industry, pilot-plant operations are now being run at pressures as high as 100,000 psi., and laboratory operations are moving into the range from 100,000 to 200,000 psi. Also, in ordnance, both laboratory studies and large-scale operations are making use of very high pressures. As pressures increase problems of equipment design become increasingly difficult. Not the least of these has been the problem of making electrical connections to the interior of a high-pressure vessel without leakage[1]. Though developed specifically for use in calibrating high-pressure standards, the Bureau's high-pressure seal may prove useful in chemical, metallurgical, petroleum refining, and other industrial equipment where high-pressure operations are monitored or controlled electrically.

Above 50,000 psi. hydrostatic pressures are commonly measured by inserting a previously calibrated resistance coil in the pressure vessel and noting the change in resistance as pressure is applied. In commercial equipment, one end of the pressure-sensitive resistance coil is usually attached to a high-pressure steel closure plug inserted in a hole in the pressure vessel. The other end of the coil is attached to an electrode which is inserted in the central opening of the closure plug but which is insulated from the plug by a bushing of some kind. External connections can then be made to the insulated electrode and the exposed portion of the steel closure plug. To minimize the problem of changes in lead resistance, a 4-lead system is usually employed for precision measurements.

In order to utilize commercial equipment insofar as possible, the

[1] "The Physics of High Pressure," by P. W. Bridgman, London, G. Bell and Sons, Ltd., 1949, pp. 51–56.

Bureau's insulating seal was designed so that the space occupied by a 4-lead connection would be the same as that required for commercially available single-wire fitting of the same shape. It was also decided that the resistance of the insulating bushing should be not less than 1000 megohms within the pressure range under consideration.

The NBS seal is composed of a soft steel electrode, a sapphire or quartz insulating bushing, a rubber O-ring, and a soft steel pad, assembled and set into a cavity in the closure plug. The soft steel pad fits directly into the plug cavity and supports the insulating bushing. Through the hole in the bushing passes the electrode, to which is soldered one end of the pressure measuring coil. The portion of the electrode external to the bushing is larger in diameter than the internal part and presents a tapered surface to the external face of the bushing. As a result of the taper, only the outer rim of this surface makes contact with the bushing unless very high pressure is applied. The rubber O-ring separates the larger-diameter portion of the electrode from the wall of the cavity in the closure plug.

In operation the seal acts in two stages. The O-ring provides the initial low-pressure seal through its compression contact with the electrode and the interior wall of the cavity in which the assembly is installed. This seal probably lasts up to about 30,000 psi. Then, at about the pressure that the O-ring becomes ineffective, the tapered section of the electrode is deformed against the bushing, thereby sealing off the leakage path between the bushing and the electrode. Simultaneously, or shortly thereafter, the soft steel pad is deformed by pressure, sealing off the leakage path between the bushing and the wall of the plug cavity.

Commercially available sapphire instrument bearings have proved quite satisfactory as insulating bushings. Besides having the required physical characteristics, they are inexpensive and can be obtained already ground to extremely exacting tolerances. It has been found that when the basal plane of the crystal becomes tangent to the planes of shear force in the loaded bushing, the crystal slips along this plane, probably because of twinning. However, this slippage does not interfere with the operation of the seal since the steel pad is again deformed against the interior surface of the bushing after slippage. Selection of bushings with the proper crystal orientation is expected to reduce the possibility of slippage. Fused quartz is less brittle than sapphire and is almost as good an insulator.

The high-pressure seal can be simplified considerably by choosing softer bushings that do not require the soft steel pad. With this type of seal, bushings of lithographic limestone have been used successfully. However, great care must be given to the selection of the limestone since this stone has many flaws which may cause the bushings to crumble under load. Once a flawless piece has been selected, it is easily turned on the lathe using sharp tools, preferably carbide, and taking small cuts.

THE FRANKLIN INSTITUTE

ANNUAL REPORT OF THE BOARD OF MANAGERS FOR THE YEAR 1954

OFFICERS AND BOARD OF MANAGERS

". . . the Officers of the Society shall be members of this Board, and to the said Board shall be confided, in general, the various objects and concerns of the Institute, with authority to devise and execute all measures which may advance its interests."

—*The Constitution of the Institute*, dated February 5, 1824

In accordance with today's By-Laws, The Franklin Institute is still governed by a Board of twenty-four Managers elected by members. The Board, in turn, elects the Institute's officers, all of whom except the Assistant Secretaries and the Assistant Treasurer are also members of the Board, *ex-officio*. The officers so elected for 1954 were:

President	S. Wyman Rolph
Executive Vice-President	Henry B. Allen
Vice-Presidents	G. H. Clamer
	James H. Robins
	Morton Gibbons-Neff
Secretary	Henry B. Allen
Treasurer	Hamilton V. Bail
Assistant Secretaries	Hamilton V. Bail
	John Frazer
	Cecil M. Waterbury
Assistant Treasurer	Cecil M. Waterbury

MEMBERSHIP

"The report of the Treasurer . . . will show you that active measures have been taken to collect the dues from the Members. . . . Hence, such of you as have not yet been called upon, are requested to prepare yourselves to pay immediately a contribution, which, small as it is at first, might by accumulating it for several years, become a burthen."

—*First Quarterly Report of the Board of Managers*, dated April 15, 1824

The Membership List for 1824 and 1825 contained the names of 621 persons. Today we are proud to report that at the end of 1954 membership totaled 6,828, as follows:

Honorary	38
Sustaining	17
Active	3,174
Associate	2,383
Student	446
Individuals with Library privileges under Company Memberships	770
	6,828

Of the 911 persons elected to membership during 1954, many joined through the efforts of members themselves. For this additional interest and help, we are deeply grateful.

With the presentation of gold keys to Frank S. Busser and C. Mahlon Kline (see Meetings Section) our list of living gold members—those having 50 years of continuous membership—totaled eighteen.

MEETINGS

"Your Board have been anxious to commence immediately the usefulness of the Institute, by the establishment of popular lectures during the present season . . . and are happy to state that the Introductory Lecture, which will be delivered on Wednesday, the 28th instant, by Dr. Robert M. Patterson, will be followed by volunteer lectures. . . ."

—*First Quarterly Report of the Board of Managers,* dated April 15, 1824

That this particular "usefulness" has continued is evidenced by the fact that the Institute held seventeen well-attended stated and joint meetings with technical societies. Our meeting rooms were used 520 times during 1954 for these and other Institute activities and by many scientific, technical, and educational organizations holding regular or special meetings at the Institute.

The older generation received special recognition at the November Stated Meeting when Frank S. Busser and C. Mahlon Kline were presented with gold keys in recognition of 50 years each of continuous membership. At the same meeting, Walter A. R. Pertuch, our Librarian, was honored for his 50 years of service with the Institute.

Nor was the younger generation forgotten in 1954. The annual James Mapes Dodge Lectures for Young People were given twice (owing to a change of date). The first series, given by Richard Sutton (Physicist, Haverford College), was entitled "Great Landmarks of Discovery in Science." Later in the year Katherine Blodgett (General Electric Company) gave a series called "The Laboratory Puts Color to Work."

The "Science Is Fun" program, sponsored by our Membership Department, had three meetings attracting members of all ages. These meetings, while more or less technical, were slanted toward the lay-

man and were all well attended. And in June more than 150 members and their friends enjoyed an informal "supper under the stars" with members of our Astronomy Division—a "Meet the Staff" program also sponsored by our Membership Department.

COMMITTEE ON SCIENCE AND THE ARTS

". . . the Board wish to call your attention to that provision of the Constitution, which declares that they shall examine all new inventions submitted to them. Your Board would have found themselves unable to do justice to this subject, unless they had adopted that By-Law which allows them to appoint Committees from the Society at large, for the examination of new machines or inventions."

—*First Quarterly Report of the Board of Managers,* dated April 15, 1824

In 1834 the name of this board of examiners was changed to the Committee on Science and the Arts; and as the merit of the Institute's judgments was recognized, we became custodian of several funds for awarding medals to those who advanced ideas for scientific applications contributing to the welfare of mankind, which are made upon the recommendation of this Committee. Twelve such medals were presented at our annual Medal Day exercises in October.

In recognition of his 35 years of service on the Committee, Joseph S. Hepburn received a George A. Hoadley Certificate.

LIBRARY

"Among the objects which your Board have in immediate contemplation, are the following . . . The formation of a Library . . . "

—*First Quarterly Report of the Board of Managers,* dated April 15, 1824

"The Library will also, it is believed, soon increase, and become valuable. The want of a proper place of deposite has hitherto checked the donations of books, and has required that those heretofore presented should be dispersed for safe-keeping, into the hands of the Members of the Committee."

—*Fourth Quarterly Report of the Board of Managers,* dated January 18, 1825

The "increase" in our Library has been phenomenal. It has grown from a shelf of books in a member's residence into a collection of 150,161 volumes, 52,736 pamphlets, and 10,856 maps at the end of 1954.

Acquisitions during the year totaled 6,390—3,896 volumes, 997 pamphlets, and 1,497 maps. We are grateful for gifts received from members and friends and for the many publications received through exchange relations between 429 societies and institutions.

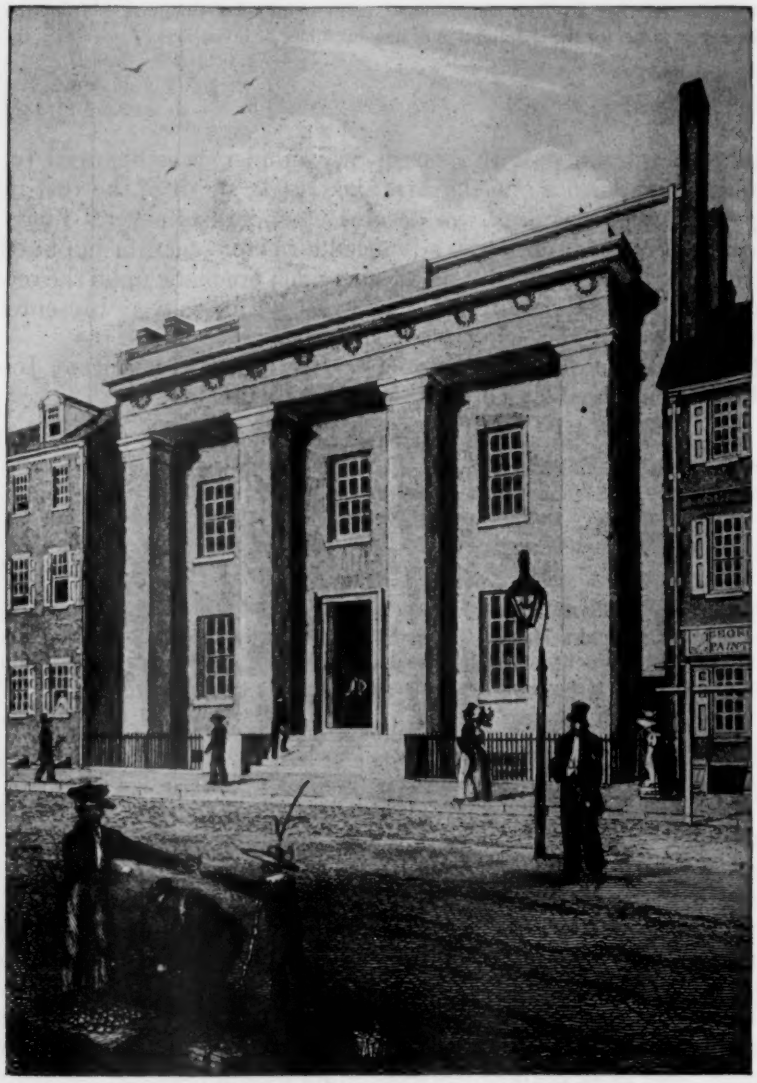
During the year approximately 7,000 persons made personal use of the Library; nearly 5,000 telephone requests for information were handled; and more than 13,000 photostats were furnished from material in the Library.

MUSEUM AND PLANETARIUM

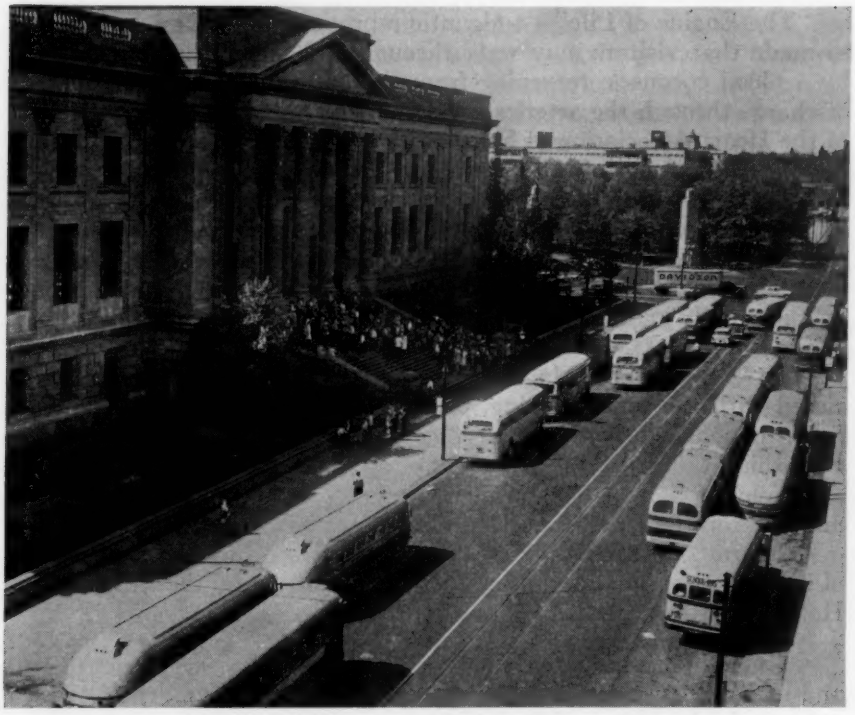
"Among the objects which your Board have in immediate contemplation, are the following . . . The creation of a Cabinet of Minerals and Models of Machines."

—*First Quarterly Report of the Board of Managers*, dated April 15, 1824

From that small beginning grew today's museum. It was in 1922 that the Board of Managers conceived the idea of an enlarged techno-



The original Franklin Institute building, as seen in 1824.



Busses bringing school children to the Museum.

logical museum. Such a museum would serve both as a supplement to the education of youth and as an agency for adult education. Unhappily the plan was beyond our resources at that time. In 1927 when a national memorial to Benjamin Franklin was proposed, the Institute was invited to take part in the preliminary meetings. When the idea of a scientific and technical museum was submitted for consideration, it was accepted as ideally designed to serve as a living memorial to the versatile Franklin. The Museum was opened to the public on January 1, 1934.

The number of visitors is proof of its popularity. Comparative figures for the past two years are:

	Paid	*Free*	*Total*
1954.....	219,360	80,001	299,361
1953.....	201,333	88,473	289,806

Our continuing effort to keep exhibits as modern and as interestingly educational as possible resulted in the construction of several new exhibits in 1954. Among those completed were:

"The Engine of Life"—a gigantic reproduction of the human heart so made that visitors may walk through and follow the course taken by a blood corpuscle returning from the body for re-oxygenation and discharge through the arteries, made possible through the co-operation of the Heart Association of Southeastern Pennsylvania and the Public Health Department of the Commonwealth of Pennsylvania;

A new "Periodic Table" of the chemical elements that furnishes fuller and more up-to-date information about the applications of the elements in life and in industry;

A short-wave "Ham" radio station, W3TKQ, expertly manned and operated daily under the sponsorship of the Haverford Township Emergency Radio Net;

The "Making of a Magazine," describing the many activities associated with the paper-making, printing, editing, and distribution of a group of magazines with a large circulation, installed by The Curtis Publishing Company;

"Folding Cartons," showing how the Robert Gair Company folds many decorative covers and containers for packaging consumer goods;

"Instruments: Servants of Industry," illustrating the growing use of instruments in industrial production, sponsored by the Minneapolis-Honeywell Regulator Company.

Although discrimination must be exercised in the acceptance of gifts and loans because display space continues at a premium, the following noteworthy items have been gratefully accepted:

A Franklin family Bible, lent by J. P. Duane, Manhasset, Long Island, New York;

Crystal and porcelain miniatures of Benjamin Franklin, given by Miss Margaret Bache, Philadelphia, Pennsylvania;

A Dynopulper for the paper-making machine, given by Rice, Barton Corporation, Worcester, Massachusetts;

Plate for printing the five-cent Confederate stamp, given by the Historical Society of Pennsylvania;

Medal of the *U. S. S. Taconic*, command ship, lent by the United States Navy Department;

Collection of firearms, given by D. I. Selfridge, Strafford, Pennsylvania;

Collection of electrical instruments, given by Princeton University, Princeton, New Jersey;

Parts of a differential analyzer, given by the Massachusetts Institute of Technology, Cambridge, Massachusetts;

Drawings from William Sellers and Company, given by R. M. Husband, Rochester, New York;

Zimphone inter-communication system, given by Herbach & Rademan, Inc., Philadelphia, Pennsylvania.

As in former years, the Museum supplemented its standard features with lecture-demonstrations, among which were the popular "House of Magic" of General Electric Company, "Energy in Action" of Westinghouse Electric Corporation, and a series of fire prevention demonstrations by the personnel of the Fire Prevention Bureau of the City of Philadelphia.

The Planetarium interrupted its normal program to stage a spectacular trip to Saturn. During Latin Week, students associated with the Classical Society were entertained at an appropriate demonstration in the Planetarium, at which time exhibits made by students to illustrate life in ancient Greece and Rome were also displayed. And the Amateur Telescope Makers completed their nineteenth year of activity with undiminished enthusiasm.

One of The Franklin Institute's Workshops for children.

The sixth annual Science Fair, held in co-operation with *The Philadelphia Inquirer*, attracted the largest number of projects of a scientific nature to date from students in local schools.

An in-service course of lectures was given by members of the staff for the purpose of instructing teachers from local schools how to make full and intelligent use of the Museum's facilities.

Young people attending the Winter Workshop were given instruction

in Astronomy, Radio-Electronics, Model-Making, Basic Chemistry, and Photography. The summer session, called "Adventures in Science," was devoted to the various sections of the Museum.

The Lunch Room, long inadequate for accommodating the large number of school children using our luncheon facilities, was re-modeled to accommodate an additional 125 visitors. At the same time, self-service machines for dispensing food were installed—to the delight of "push-button" fans, young and old.

RESEARCH LABORATORIES

"There are objects of still greater importance, which the Board entertain a hope may at some future day be within the means of the Society; foremost among these would be the establishment of an Experimental Workshop and Laboratory . . . but this would require a much larger sum of money than the Society has at present at its disposal."

—*First Quarterly Report of the Board of Managers,* dated April 15, 1824

While waiting for the formal "Experimental Workshop and Laboratories," researches in the public service were undertaken as the occasion required by groups of members who conducted investigations for industry. Included were researches on Water Wheels, Steam Boilers, Weights and Measures, Gas Lighting and Highways, and Screw Threads Standards.

While organized at the start of World War II, the laboratories began research on a formal basis for industry in 1946 under the name The Franklin Institute Laboratories for Research and Development.

Today, both industrial and governmental research continues at a high level, consistent with available space. Among the Armed Services, we are well recognized for our research work; and the growth of industrial research is evidence that our Laboratories are becoming even better known to industry.

In 1954 the general plan for development of the Laboratories' services was to promote industrial support for research, while holding government support at levels determined by available space and facilities. The result: a 55 per cent increase in revenue from industrial projects, and a slight increase in total revenue. The table below summarizes these statistics for the last three years:

	1954	*1953*	*1952*
Revenue from Industrial Projects	$ 714,569.	$ 459,818.	$ 252,054.
Revenue from Government Projects	2,185,047.	2,401,465.	$2,335,583.
Total Revenue	$2,899,616.	$2,861,283.	$2,587,637.

Two aspects of our industrial business deserve special mention: co-operative (or multiple-sponsor) research and diversification of our

clientele. Co-operative research, where several companies combine to support a research project they could not undertake separately, are becoming an important vehicle for our services. Ten projects of this sort are current, including the National Asphalt Research Center, which was the first. In this category are five projects we are conducting for trade associations, included because they are so similar in nature.

The other aspect—diversification of clientele—is best seen in a review of contracts received in 1954. Of the 66 projects begun for industry in that year, 27 were for companies that had not previously sponsored work and 30 companies entered our list of clients for the third, fourth or fifth time. Interesting, too, is the fact that 23 per cent of our total industrial revenue came from companies outside the manufacturing field, such as utilities and foundations. We are gratified that we have been receiving grants from several foundations in support of certain projects. Of the 15 categories of industry-supported projects in 1954, the following contributed 65 per cent: petroleum and coal products, electrical machinery, primary metals, machinery (excluding electrical), and chemicals and allied products.

Space permits mentioning only a few of the projects completed during 1954:

A two-year project for the Association of American Railroads in which our staff evaluated many bearing materials for possible use in freight cars, and uncovered information that might lead to eventual reduction in the frequency of hot-boxes;

An automatic tuning device for radio receivers which will, when a button is pushed, scan the radio frequency band automatically, stopping when it encounters a radio signal of sufficient strength;

The A-C Network Calculator for seven power companies, which was installed, tested, and put into operation in June;

A comparative study of various facsimile transmission systems, which would allow a bank to maintain only one central file of specimen signatures while providing rapid service to its branches;

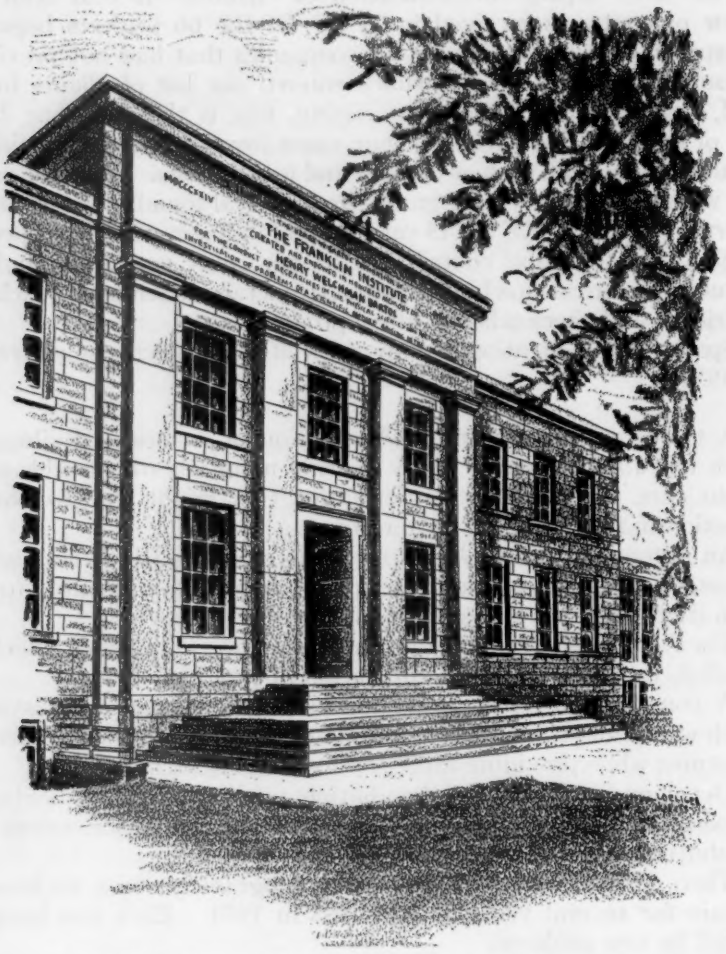
Development of a special radiation pyrometer for measuring the temperature of short-circuiting segments of frequency converters without shutting down the machine;

Three air traffic control projects—a program in which we have had a share for several years—terminated in 1954. Each has been succeeded by new projects;

The first phase of a project under which we are seeking a method for detecting cavities beneath city streets—from the street surface.

The year 1954 saw the addition of 3,634 square feet of space (a roof pent-house) when the A-C Network Calculator was completed; and at present the Laboratories occupy 77,267 square feet. New research

equipment worth $40,000 was purchased, the main items being an ADL Collins helium cryostat for studies in low-temperature physics, an analytic feedback balance for measuring magnetic susceptibility, and an Ecco high-frequency furnace for zone-melting studies. In our shops, we completed work on an infrared spectrometer and began construction of a five-horse power journal testing machine.



The Bartol Research Foundation building in Swarthmore, Pa.

Late in 1954, a new section—Heat and Power—was established within the Mechanical Engineering Division. The staff of this section will develop apparatus for thermal power systems (turbines, heat exchangers, rockets, etc.) and will perform research in the fields of thermo-

dynamics, heat transfer, thermal stresses and the properties of liquids and vapors, and matters associated with atomic energy for power development.

The Bartol Research Foundation, Swarthmore, Pennsylvania:

In 1918 Henry W. Bartol, prominent Philadelphia industrialist, bequeathed his residual estate to The Franklin Institute. As inscribed over the entrance of the main building the laboratories are devoted to "the conduct of researches in the physical sciences and for the investigation of problems of a scientific nature arising in the industries." The Bartol Research Foundation is located both on the campus of Swarthmore College and adjacent Springfield Township.

In 1954 the Foundation continued what is a long-range series of projects covering cosmic rays and nuclear physics, with particular relation to neutron scattering, the scattering of other nuclear particles, radioactivity, and allied researches. Solid-state researches are in progress on electrical and optical properties of ionic crystals, bombardment effects by high speed electrons and by nuclear particles, phosphorescence and energy storage.

The staff of the Foundation has been enriched by the addition of new personnel; and new instrumental equipment, constructed largely in Bartol's shop, has been added. Also an efficient control room for the operation of the generators has come into being and is now in continuous service.

The Biochemical Research Foundation, Newark, Delaware:

The Institute's research activities were extended in 1935 when it assumed trusteeship of The Biochemical Research Foundation, where the staff actively engages in research on problems of cell growth, both normal and cancerous.

During 1954 the study of substances that change the use of glucose was extended. Screening of substances which may act as inhibitors at one or more stages of the break-down of sugar continued in three directions. Culturing of certain tumors of mice (tissue culture technique using tubes or flasks) progressed in a slow but satisfactory manner. Tumor cells were grown for as long as three days, and investigation of the various conditions indicated a longer growth period is not impossible. The technique of incubation of freely suspended non-solid tumor cells upon the inner membrane of the embryonic chick was also studied. Partial success has been obtained along this line of research.

A long-term study of the relation of certain diets to growth and ultimately to precancerous or cancerous conditions was started. Nearly 900 rats are being studied in order to relate enzyme response and dietary control with growth, normal and cancerous.

The Biochemical Research Foundation building in Newark, Del.

Some valuable data were accumulated on the effect of external radiation (X-ray and ultraviolet) upon the non-solid tumor cells. Growth curves were plotted, and these will be used to determine the efficiency of the scintillators which emit the secondary radiation that is lethal to living cells. In this study co-operation continues between The Biochemical and Bartol Research Foundations.

JOURNAL OF THE FRANKLIN INSTITUTE

"Perhaps a measure of equal importance would be the publication of a Journal. . . . No doubt can exist that such a Journal would diffuse among our mechanics, at a low rate, much useful information."

—*First Quarterly Report of the Board of Managers,* dated April 15, 1824

In August, 1825, a prospectus was issued announcing the forthcoming publication of THE FRANKLIN JOURNAL AND MECHANICS' MAGAZINE "to diffuse information on every subject connected with the useful arts." In 1828 the name was changed to the JOURNAL OF THE FRANKLIN INSTITUTE.

In line with the current trend in scientific publications, the JOURNAL cover was re-designed in 1954 to include titles of papers published therein. During the year, 36 technical papers and 12 papers of general scientific interest were published. Manuscripts submitted for publication totaled 74.

Circulation for the year increased slightly to 4,748; and the JOURNAL was received in 54 foreign countries and in 47 states in this country.

In preparation for the 250th Anniversary Celebration of Franklin's birth in 1956, invitations were issued to the societies and institutions with which Franklin had been connected, asking them to prepare papers for a special issue in January 1956. Nineteen affirmative replies were received, ensuring the success of the project. The issue will stress progress made during the past 250 years in those fields in which Franklin himself was active.

PERSONNEL

Behind all of this activity are the employees of The Franklin Institute. As of December 31, 1954, staff members numbered 506 (as compared with 552 at the end of 1953). During the year ten employees left to serve in the armed forces and three returned to the employ of the Institute after having completed their tours of duty.

Elizabeth Armstrong (Matron) retired after 21 years of faithful service, William Elwood (Captain of Museum Guards) after 20 years, and William Byers (Museum Guard) after 13 years.

Death claimed two loyal employees: Alexander McWade (Museum Guard for 19 years) and Harry Lodge (Maintenance Plumber for 7 years). Their passing is recorded with deep sorrow.

PUBLIC RELATIONS

Our Public Relations department continued its efforts to make the public aware of the Institute, its activities and its functions, through the media of press, radio and television.

A total of 148 radio programs, both local and network, were presented by or for the Institute. Included was "Science, Servant of Man," a weekly feature presented the latter part of the year over the educational station WHYY.

Local and network television programs totaled 36, including the weekly "Horizons Unlimited" featuring W. F. G. Swann, Director of the Bartol Research Foundation and Senior Adviser to our Laboratories for Research and Development.

The number of column inches of newspaper stories and pictures concerning The Franklin Institute was most gratifying. This is a rough method of determining the amount of public notice given to the Institute and its activities.

Nation-wide notices in press and on radio and television were given special events outlined in preceding sections of this report.

The Institute News continued to expand its circulation, keeping pace with the advance in membership and answering requests from other interested sources.

Again, hundreds of inquiries about Franklin, his interests, and the various phases of operation of the Institute were answered. And the number of speeches by staff members before scientific, industrial and civic groups was gratifying.

HOSTESS COMMITTEE

The Hostess Committee continued to earn our gratitude for its interest in the Institute. Continuation of the grounds-improvement program was the Committee's main project for 1954. Hostesses participated in the receptions preceding dinners and lectures and contributed much to their success.

ENDOWMENTS

With the reorganization of our fund-raising activities early in 1954, emphasis has been placed on increasing funds to help meet our operating expenses, although at the same time other fund-raising efforts have been carried out.

The group of public-spirited individuals and companies known as "Friends of Franklin" continued its generous support of the Institute with contributions of $37,355. We are gratified to report an increase in both members and contributions over 1953. We are even more gratified by the enthusiastic assistance received from members and friends of the Institute who gave both time and financial support to this effort and by the sympathetic response received to our appeal for funds for our program to advance scientific interest and education.

Friends and associates of the late Philip C. Staples, our president from 1937 to 1941, contributed $8,000 to establish an annual lecture at the Institute in his honor. (The balance of $2,000 needed to reach the goal of $10,000 for this fund was received early in 1955.)

At our invitation, a group of lawyers and trust officers of banks in the Philadelphia area visited the Institute in October to see at first hand the work of the Institute and its need for funds to perform its many educational and scientific services. The enthusiasm of this group was encouraging.

There is a continuing and pressing need for contributions, endowments and bequests in order to maintain our educational and scientific program that has proved so useful to schools, industry, and the general public.

FINANCIAL STATEMENTS

The accounts of the Institute have been audited by Messrs. Lybrand, Ross Bros. & Montgomery whose report is available to members for examination in the office of the Treasurer. The following balance sheet and statement of income and expenses have been summarized and adapted from that report. It will be noted that figures for the Bartol Research Foundation, a special fund of the Institute, are included in these statements.

BALANCE SHEET AT DECEMBER 31, 1954
AND DECEMBER 31, 1953

Assets

	Dec. 31, 1954	*Dec. 31, 1953*
General Operating Funds		
Cash	$ 318,915	$ 268,814
Amounts receivable from laboratory research contracts	495,692	470,186
Other receivables	33,577	59,797
Inventories	44,243	48,592
Deferred charges	17,557	20,263
Total general operating funds	$ 909,984	$ 867,652
Bartol Operating Funds		
Cash	55,866	84,728
Accounts receivable	42,342	32,215
Inventories	22,799	13,207
Deferred charges	2,066	1,997
Total Bartol operating funds	$ 123,073	$ 132,147
Endowment Funds		
Investments at cost (market values $6,710,743 and $5,641,325 respectively)	4,644,937	4,450,858
Uninvested principal cash	218,020	54,657
Advances to general operating funds	51,000	51,000
Advances to general plant and property funds		45,000
Income cash	22,733	23,613
	$ 4,936,690	$ 4,625,128
DeMonclos Trust Fund:		
Investments at cost	178,547	167,009
Uninvested principal cash	(109)	450
Income cash	2,672	2,369
	$ 181,110	$ 169,828
Total endowment funds	$ 5,117,800	$ 4,794,956
General Plant and Property Funds		
Cash	3,091	25,451
Buildings, equipment, fixtures, etc. (less reserves for depreciation of $1,326,609 and $1,170,213, respectively)	4,019,551	3,925,966
Advances to general operating funds	446,275	386,176
Total general plant and property funds	$ 4,468,917	$ 4,337,593
Bartol Plant and Property Funds		
Land, buildings, equipment, fixtures, etc.	729,106	689,748
Costs of generator in excess of amount reimbursable from U. S. government		99,702
Total Bartol plant and property funds	$ 729,106	$ 789,450
Total assets	$11,348,880	$10,921,798

Liabilities and Funds

	Dec. 31, 1954	Dec. 31, 1953
General Operating Funds		
Accounts payable, accrued expenses, etc.	$ 263,301	$ 283,385
Deferred income	43,236	45,029
Amount due to plant and property funds	446,275	386,175
Amount due to endowment funds	51,000	51,000
Capital stock	10,430	10,430
Operating fund balance	95,742	91,633
Total general operating funds	$ 909,984	$ 867,652
Bartol Operating Funds		
Accounts payable and accrued expense	16,842	25,316
Operating fund balance	106,231	106,831
Total Bartol operating funds	$ 123,073	$ 132,147
Endowment Funds		
General and restricted endowment funds and funds functioning as endowment including those of the Bartol Research Foundation	4,913,957	4,601,515
Unexpended and undistributed income	22,733	23,613
	$ 4,936,690	$ 4,625,128
DeMonclos Trust Fund:		
Principal	178,438	167,459
Undistributed income	2,672	2,369
	$ 181,110	$ 169,828
Total endowment funds	$ 5,117,800	$ 4,794,956

	Dec. 31, 1954	Dec. 31, 1953
General Plant and Property Funds		
Contributed funds	4,468,917	3,952,309
Funds designated for special equipment		304,403
Amount due to endowment funds		45,000
Amount due for special equipment		35,881
Total general plant and property funds	$ 4,468,917	$ 4,337,593
Bartol Plant and Property Funds		
Plant and property fund balance	729,106	689,748
Reserved for generator costs in excess of reimbursable amounts		99,702
Total Bartol plant and property funds	$ 729,106	$ 789,450
Total Liabilities and Funds	$11,348,880	$10,921,798

COMPARATIVE STATEMENT OF INCOME AND EXPENSES
FOR THE YEARS ENDED DECEMBER 31, 1954 AND 1953

	1954	*1953*
Operating Revenue		
Membership dues	$ 62,717	$ 62,573
Museum admissions and other museum income	150,563	136,361
Laboratory research projects	2,899,616	2,861,283
JOURNAL income	22,915	24,236
Other activities	15,739	15,582
	$ 3,151,550	$ 3,100,035
Operating Costs and Expenses		
Museum	362,153	352,586
Library, net of amounts transferred to other departments	78,488	80,700
Laboratory research projects	2,751,807	2,677,368
JOURNAL expense	31,712	34,158
Administrative and general expenses, net of amounts transferred to other departments	86,955	98,005
	$ 3,311,115	$ 3,242,817
Operating loss, before other income and charges	$ 159,565	$ 142,782
Other Income		
Income from endowment funds	98,746	89,673
Donations from Friends of Franklin and other gifts	38,047	24,648
Appropriation, Commonwealth of Pennsylvania	46,500	44,771
Other income	17,043	14,848
	$ 200,336	$ 173,940
	$ 40,771	$ 31,158
Other Charges		
Cafeteria Expense	5,957	6,158
Interest	2,349	2,880
Miscellaneous	423	459
	$ 8,729	$ 9,497
Excess of income transferred to Operating Fund Balance	$ 32,042	$ 21,661

Respectfully submitted,

By order of the Board of Managers,
S. WYMAN ROLPH,
President

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, June 8, 1955.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JUNE 8, 1955.

MR. RALPH H. MCCLARREN in the Chair.

The following reports were presented for final action:

No. 3290: Ballantine Medal.

This report recommended the award of the Stuart Ballantine Medal to Claude Elwood Shannon, of Chatham, New Jersey, "In consideration of his recognition of communication as essentially a statistical process, his identification of the elements of communication systems with the appropriate statistical functions, and his welding of the powerful methods of mathematical statistics into a comprehensive theory of communication which permits precise and rapid evaluation of proposed new communication systems, and points the way for significant new developments."

No. 3291: Henderson Medal.

This report recommended the award of the George R. Henderson Medal to Carleton K. Steins, of Merion Station, Pennsylvania, "In recognition of his numerous outstanding inventions contributing to the progress and efficiency of the railroad transportation systems, and his high engineering attainments in this same field."

JOHN FRAZER,
Secretary to Committee

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, JUNE 15, 1955

ACTIVE FAMILY

John P. Bracken
Richard Stone Doughty
John M. Huebner
John P. McArthur
H. Edward Slough

ACTIVE

Charles P. Bailey
Raymond Cannon
Louis Charles Eskin
John Gilchrist, Jr.
John M. Hauserman
J. W. Humrichouse
Leslie P. Hunneman
Walter B. Keighton, Jr.
Raymond F. Kroll
Lawrence T. LaPatka
Howard W. Leadbeater
W. H. Mapp
Robert M. Mattox
John McFarland
I. Jerome Milgrim
John B. Plumb
H. Arthur Porter
Mrs. Edwin M. Rhea
Henry F. Sachsenmaier
Otto F. Schoenhut
Malcolm G. Shoemaker
D. Bobb Slattery
A. T. Steelman
John D. Wagner
Allen H. Wetter
W. Dewees Yeager

ACTIVE NON-RESIDENT

James F. Bieret, Jr.
Samuel T. Hollingsworth
John D. Wise

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

TUSKA, C. D.: Increasing the Creativeness of Engineers.
KLEIN, BERTRAM: Certain Aspects of Efficient Structural Design.
BRUCE, CHARLES E. R.: Note on "Upward Stepped Leaders from the Empire State Building."
CHENG, DAVID K. AND PRANAS GRUSAUSKAS: Determination of Aperture Phase Errors in Microwave Reflectors.
KROLL, W. J.: How Commercial Titanium and Zirconium Were Born.
MOON, PARRY AND DOMINA EBERLE SPENCER: On Electromagnetic Induction.
HONNELL, PIERRE M. AND ROBERT E. HORN: Matrices in Analogue Mathematical Machines.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A. M. until 5 P. M. on Mondays, Tuesdays, Thursdays, and Fridays; 2 P. M. until 10 P. M. on Wednesdays; and 9 A. M. until 12 NOON on Saturdays.

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TO MEMBERS AND THOSE WHO KNOW THE FRANKLIN INSTITUTE

All of us associated with The Franklin Institute are gratified by the support given by its benefactors in the past. Without their generosity our privately endowed Institute would have been unable to serve industry, the professions, and the public as it has and should.

There is still a continuing and pressing need of contributions and bequests —both for general and specific endowment, and of income to be judiciously used. Your response will be most earnestly appreciated.

Checks should be made payable to The Franklin Institute, and sent to The Franklin Institute, Benjamin Franklin Parkway at 20th Street, Philadelphia 3, Pennsylvania.

Where property is transferred, title should be in the name of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts.

We shall be glad to supply any additional information regarding gifts and memorials. Please write to the above address, attention of the Endowment Department.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES
FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

FERROMAGNETISM IN NOBLE METAL ALLOYS CONTAINING MANGANESE

BY

F. A. OTTER, JR., P. J. FLANDERS AND E. KLOKHOLM

Even though pure metallic manganese is closely allied to the ferromagnetic family of iron, cobalt, and nickel, it is paramagnetic to temperatures as low as $-259°$ C. Therefore, at ordinary temperatures, manganese never attains the magnetic "strength" of iron, cobalt and nickel. Theoretically the existence of ferromagnetism in solids depends on a quantity known as the mutual magnetic exchange energy. This in turn depends on the ratio of the distance of closest approach of two atoms in the solid to the radius of an incomplete inner shell ("3d" shell) of electrons in the atom. If this ratio of distances is greater than 1.5 the mutual exchange energy is positive and the solid is ferromagnetic; if the ratio is less than 1.5, the energy is negative and the solid is not ferromagnetic. For iron, cobalt and nickel it is greater than 1.5 and for manganese it is less than 1.5; the former are then, as is well known, ferromagnetic, while the latter is not.

From the preceding, one might conclude that by increasing the separation between manganese atoms, the ratio of atomic separation to "3d" radius could be altered favorably, and the result would be a ferromagnetic material containing manganese. Atomic separation of this kind can be accomplished by alloying manganese with a non-magnetic element. Ferromagnetic ternary alloys of copper, aluminum, and manganese (Heusler alloys) are typical examples of the existing magnetic manganese alloys and compounds.

Dilute binary noble metal-manganese alloys (i.e., alloys containing a small amount of manganese as the solute metal) have not been widely reported. However, this laboratory has been conducting an investigation of the thermoelectric properties of dilute noble metal-transition metal alloys. During this work, non-linear deviations in the temperature dependence of the electrical resistivity, characteristic of ferromagnetic metals and alloys, were observed for those containing manganese. Normally this dependence for dilute alloys is linear over a fairly wide temperature range. For noble metal alloys containing more than 8 at. % Mn the deviations occur as marked changes in

slope of the curve of electrical resistivity vs. temperature. The temperature at which the change in slope takes place increases with increasing manganese content.

The effect is caused by the influence of the loss of ferromagnetism on the electrical resistivity at the Curie temperature. Subsequent magnetic measurements on the alloys of Au + 18.64 at. % Mn, Au + 13.01 at. % Mn and Ag + 14.58 at. % Mn confirmed the expectation that the alloys were ferromagnetic with Curie temperatures of 317° K, 139° K, and 163° K. The Curie temperature increased with increasing manganese content, while in the same alloy system the magnetic moment per manganese atom decreased with increasing manganese content.

Further investigations of these alloys are planned to correlate electrical and magnetic properties, particularly at low temperatures.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, FOUNDER
WILLIAM G. BATT, ACTING DIRECTOR

Krebs Ascites Carcinoma Inorganic Phosphorus Content and Phosphatase Activity.—J. O. ELY. While making investigations on the Krebs ascites carcinoma, it was found convenient to measure the phosphatase activity and the inorganic phosphorus content of the tumor cells and of the tumor ascitic fluid. The ascitic fluid containing the Krebs ascites tumor cells was obtained from white Swiss mice 6–11 days after the mice were inoculated. Grossly hemorrhagic ascitic fluid was avoided. The fluid was centrifuged for 5 minutes at 1000 G and the supernate removed for inorganic phosphorus and phosphatase determinations.

The sedimented cells were washed 3 times by suspension in 0.85% sodium chloride solution and were then suspended in a volume of 0.85% sodium chloride solution equal to the volume of the supernate. The leucocytes which separated from the tumor cells during washing were discarded.

The incubation procedure of the Bodansky method was used for alkaline phosphatase with modification to permit use of the method of Fiske and SubbaRow (*J. Biol. Chem.*, **66**, 375, 1925) for the determination of phosphate liberated; for acid phosphatase the conditions described by Shinowara, Jones and Reinhart (*J. Biol. Chem.*, **142**, 921, 1942) were followed, likewise modified to permit use of the Fiske and SubbaRow phosphate method (P. B. Hawk, B. L. Oser, and W. H. Summerson, "Practical Physiological Chemistry," 13th Edition, The Blakiston Company, Inc., New York, 1954, pp. 630 ff.). Inorganic phosphorus of the supernate and the cells was determined by the Fiske-SubbaRow method.

The inorganic phosphorus content of the supernate of the ascitic fluid from 12 mice was 11.5 ± 0.54 mg. per 100 ml. The amount per 100×10^6 ascitic cells was 0.146 ± 0.008 mg. (24 mice).

The phosphatase activity of the cells is expressed in terms of the number of cells required to liberate 1 mg. of phosphorus in 1 hour at 37°–38° C. from β-glycerophosphate substrate at pH 9.1 for alkaline phosphatase, and at pH 5 for acid phosphatase. The number of cells required was: for alkaline phosphatase $1889 \pm 105 \times 10^6$ (14 mice), and for acid phosphatase $1273 \pm 79 \times 10^6$ cells (14 mice).

The phosphatase activity of the supernate of the ascitic fluid in terms of mg. of P liberated per hour was: alkaline phosphatase 0.57 ± 0.02 mg. (7 mice); acid phosphatase 1.2 ± 0.12 mg. (8 mice). The

activity of pooled samples of serum of these mice was 1 mg. for alkaline phosphatase and 1.35 mg. for acid phosphatase.

Since the data have not been expressed on the weight basis, comparison of the phosphatase activity of the ascitic cells can not be made with that of other tumor cells. The alkaline phosphatase activity of the Krebs ascites tumor cells, however, was approximately 1–2 per cent of that shown by rabbit leucocytes (D. M. Cram and R. J. Rossiter, *Can. J. Research*, **E**, **27**, 290, 1949). The acid phosphatase activity was about 50 per cent of that shown by rabbit leucocytes.

The supernate of the ascitic fluid was similar to mouse serum in inorganic phosphorus content and acid phosphatase activity, but lower in alkaline phosphatase activity.

BOOK REVIEWS

LAPLACE TRANSFORMS FOR ELECTRICAL ENGINEERS, by B. J. Starkey. 279 pages, 15 × 22 cm. London, Iliffe & Sons, Ltd., 1954. Price, 30 s.

The title of this 280-page book evokes vague associations with cook-book type treatises of the variety of "Mathematics for. . . ." However, the feeling of apprehension is quickly allayed by reading the first few sentences of the author's text. In fact, this reviewer soon got engrossed in the author's presentation of the subject familiar to him, and read on with a mounting feeling of admiration for the author's didactic skill.

This book is an outgrowth of a series of lectures on the Laplace transformation given by the author to his colleagues at the Signals Research and Development Establishment in Great Britain. It is addressed to electrical engineers whose knowledge of higher mathematics is limited to the differential and integral calculus, the principles of the theory of complex numbers, and rudiments of vector analysis, but who are acquainted with the "symbolic" of "vectorial" method of analysis of alternating currents.

The first chapter contains a brief, but excellent, exposition of the reasoning underlying the symbolic methods and the associated Fourier series and Fourier integral techniques. Drawing upon these familiar concepts for analogies, the author, then presents the formalism of the Laplace transformation. He introduces the reader immediately to the special, but practically important, type of system functions represented by the quotient of two rational polynomials and explains the use of the method of partial fractions in determining inverse Laplace transforms in this case. The application of these formal rules to electric circuit theory is illustrated in great detail in Chapter 4, with emphasis on the physical significance of the location of the poles of the system function at hand.

Chapters 5 to 8 and Appendix A are devoted to an exposition of the elements of the theory of functions of a complex variable. They are truly remarkable for the didactic skill with which the essential concepts are presented. The author's use of the representation of a complex function as a vector field in the plane of complex numbers contributes greatly to the clarity of exposition of the process of contour integration. These chapters could be read with profit by every student of the theory of functions of a complex variable, regardless of its bearing on the principal subject of this book.

Provided with the theoretical background sufficient to handle meromorphic functions, the reader is introduced into the theory of the Laplace transformation in a more rigorous manner. The familiar scale-change, shift, attenuation, product, final-value, initial-value, and other theorems are given and illustrated by practical examples. Again, the reader's understanding of the physical significance of these operations is aided by skillfully chosen graphical representations. The "unit step," "square pulse," and "unit impulse" functions, the principle of superposition and Duhanel's theorem are discussed at some length. The application of the Laplace transformation in the solution of linear partial differential equations is briefly indicated.

Chapter 11 contains a very useful collection of examples of different uses of Laplace transforms and of various methods of finding the inverse Laplace transforms. Chapter 12 presents the extension of the theory of the Laplace transformation to polymorphic functions. Appendix B gives definitions and some of the properties and series representations of some of the higher transcendental functions frequently occurring in practical problems. Appendix C is a table of Laplace transforms with 166 entries.

In all, this book is eminently well suited as a text-book for an introductory course on Laplace transformation for budding, or full-fledged, electrical engineers. It may as well be studied with profit by applied mathematicians working in other fields. This powerful analytical tool may be used to great advantage in the solution of many problems outside the realm of electrical circuit theory.

OTTO R. SPIES

THE DEVELOPMENT OF THE CONCEPT OF ELECTRIC CHARGE, by Duane Roller and Duane H. D. Roller. Harvard Case Histories in Experimental Science, No. 8. 97 pages, 15 × 23 cm. Cambridge, Harvard University Press, 1954. Price, $1.60.

We have had occasion to praise the method of teaching the history of science advocated in the essays in this series of "Harvard Case Histories in Experimental Science." The latest addition to the series is indicative of the excellence of the method, since it is the work of two students who express their gratitude to the former general editor of the series, Dr. James B. Conant, for having made possible their participation in the course utilizing the case history method.

This work deals with the history of electricity to the time of Coulomb, or before the introduction of quantitative methods into the electrical science. It is, therefore, not in any degree a technical work, as it treats with the speculations and experiments of those natural philosophers who laid the foundations of the science. This is in keeping with the spirit of the series of which it is a part. For the purpose is to treat the evolution of scientific concepts from the educated layman's viewpoint, so that he may be assisted to relate the developments in the natural sciences to those in other fields of human activity.

The present age is sometimes called the Scientific Age. This does not mean that every member of the community is an expert scientist—far from it. It does mean, however, that the labors of scientists have given the age certain features which influence the life of every citizen to some degree. Accordingly, it is desirable that as many as possible should have some understanding of the scientists' work, of their aims, their point of view, and their methods. The series offers readers an opportunity to learn of the parts played by the early scientists in advancing man from a condition of ignorance toward knowledge and toward the mastery of nature and himself.

The newest addition to the series accomplishes its object within its defined field. Giving copious extracts from the original texts, the authors present a picture of the electrical science as it evolved at the hands of experimenters whose knowledge was handicapped by the restrictions imposed upon them by the general level of knowledge at their times. The reader perceives how discoveries of fundamentals were made without benefit of that apparatus which is a commonplace in college laboratories of today. The contributions of the great men in the early stages of electrical science, Gilbert, Hauksbee, Stephen Gray, Abbè Nollet, Benjamin Franklin, and many men of lesser reputation are adequately presented. We feel that the work of Otto von Guericke might have been noted, for his invention of the static electrical machine undoubtedly furnished the starting point for many another experimenter, and he was the first to publish observations of the conduction of electricity along a linen thread.

The little book is up to the standard set by the others of the series, which means it is a very useful introduction to the subject for anyone who is not a specialist.

T. COULSON

TABLES OF INTEGRAL TRANSFORMS, VOLUME 2, edited by A. Erdélyi. 451 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $8.00.

This volume of 451 pages completes the work of compilation of tables of integral transforms undertaken by the Bateman Manuscript Project of the California Institute of Technology in 1946. The previously published first volume, comprising Chapters I to VII of this work, contains tables of the closely inter-related Fourier, Laplace and Mellin transforms. The present volume, Chapters VIII to XX, contains in the first four chapters tables of "Bessel transforms," comprising the closely inter-related J-(Hankel-), Y-, K-, and H-transforms. A short table of Kontorovich-Lebedev transforms forms Chapter XII. Chapters XIII to XV are formed by tables of "Fractional Integrals" and of Stieltjes and Hilbert transforms. The former provide the means for evaluation of integrals and derivatives of fractional order.

Chapters XVI to XX contain integrals of higher transcendental functions not listed in the tables of Chapters I to XV. These include the Tchebichef, Legendre, Gegenbauer,

Jacobi, Hermite and Laguerre polynomials; the gamma, the incomplete gamma, and related functions; as well as the Legendre, Bessel and Hypergeometric functions.

An appendix of 24 pages gives the notation used and definitions of the higher transcendental functions included in this work. An index of notations appears at the end of the volume.

The publication of this volume will be welcomed by pure and applied mathematicians and by enlightened engineers, whose problems involve functions included in this work. Everyone who uses the methods of functional analysis will appreciate the great saving of time and mental effort afforded by a compilation of the explicit forms of many special cases of a general relationship. The editors of this monumental work are to be congratulated for the systematic manner in which the material is arranged and cross-correlated.

OTTO R. SPIES

CATALYSIS, VOL. 2. FUNDAMENTAL PRINCIPLES (PART 2), edited by Paul H. Emmett. 473 pages, diagrams, 16 × 24 cm. New York, Reinhold Publishing Corp., 1955. Price, $12.00.

The word *catalysis* has fundamental implications. From our very process of life to the performance of modern oil refineries, catalysts are among the basic ingredients needed to perform rapidly and economically the right kind of chemical reactions. Present-day importance of practical catalysis has served to emphasize the need for fundamental knowledge of catalysts behavior. As scientific research broadens this knowledge and a large body of experimental data is accumulated, the need arises for comprehensive reviews of theory and practice of catalysis. This is the aim of the book under review. This aim has been excellently fulfilled, and, as Vol. 1, which appeared in 1954, the present volume maintains a high level of presentation. Throughout the six chapters, which make up the book, the fundamental role of the catalyst is viewed from different standpoints.

Thus, the first article by W. B. Innes summarizes literature data on gas phase catalytic reactions. References are reported for eight groups in which catalytic reactions have been classified: acid catalyzed, hydrogenation-dehydrogenation, oxidation, hydration-dehydration, halogenation-dehalogenation, dehydrogenation-dehydration, isotope exchange, and miscellaneous reactions. It is probably to regret that the collection of data has been stopped at 1952, thus making the article slightly out-dated, as compared with the other articles of the book whose references are well into 1953 and 1954. In the second paper A. Wheeler discusses how mass and heat transfer in the catalyst pores can influence the rate and activation energy of the catalytic reaction. This is an important aspect of catalytic phenomena, which is often overlooked in fundamental work on porous catalysts. A description of the practical methods available to determine the pore structure of catalysts is given. A few simple tests aimed at determining the effect of mass transfer and temperature gradient on catalyst performance are presented. It is also shown how catalyst selectivity and poison characteristics can be affected by the pore structure of the catalyst.

In the following two articles H. M. Hulburt presents the basic formulation of solid state theory as applied to metal and semiconductors and reviews the suggestions and speculations which have been put forward to explain the nature of surface complexes. These articles clearly show how rudimentary and incomplete is our present understanding of solid surfaces, as compared to our knowledge on the structure of the bulk phase. It is therefore quite timely that we start to apply this knowledge to explaining surface properties. A good deal of work on the subject discussed by these two articles has been recently carried out by the Hauffe's school in Germany. However, no mention of this and some other recent contributions on the same subject is made in these articles.

The fifth paper by R. B. Parlin, M. B. Wallenstein, B. J. Zwoliuski and H. Eyring deals with the application of the absolute rate theory to heterogeneous catalysis. The thermodynamics and the statistical mechanical treatment of adsorption is discussed, emphasizing the point of view of the Eyring school. A review of the concept of the activated complex, and its application to the catalytic hydrogenation of ethylene, and a

discussion on the nature of the catalyst are included.

The final paper by E. L. King discusses catalysis in homogeneous liquid phase: kinetics, mechanism, medium effects, acid-base and negative catalysis. This is a good review of the subject, which has had the privilege in recent years of a relatively large number of review articles.

On the whole the Editor should be commended for having been able to present, bound in this volume, an authoritative series of papers, which, together with those of volume one, clearly fulfill a role in present-day catalytic research. G. PARRAVANO

MAGNETIC AMPLIFIERS, by Herbert F. Storm. 545 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1955. Price, $13.50.

A great deal of information has been published about the magnetic amplifier; however, the greatest portion of this material presupposes a knowledge of the subject. Each of the works which comprise this store expounds on but a small sample of the magnetic amplifier's characteristics. Thus, there exists a collection of works which is, in effect, discontinuous because each book presents the viewpoint of its author. Because of the various approaches coupled with different points of emphasis and in some cases the absence of interlocking works, the different authors may sometimes appear to be in conflict.

In this latest addition to the General Electric Series, Dr. Storm and his associates have filled a void which has long existed in this field. Compiled in this volume is a complete treatment of the magnetic amplifier. It covers not only the theory but the practical applications as well. Starting with the modern theory of magnetism, the authors continue through an analysis of the device's complex operation. This work is terminated in a discourse on the numerous applications of the magnetic amplifier.

Since the magnetic characteristic of the core material is the amplifier's heaviest weighted parameter, considerable detail is expended on this subject. An interesting similitude is depicted in the first section; the effects of variations in temperature, frequency, lamination thickness, etc., are displayed as changes in the static B-H curves. Through this novice, variations in a magnetic amplifier's operation with changes in the above factors are readily accountable.

Because of the nonlinear nature of the magnetic amplifier and the dependence of its operation upon these nonlinearities, it can not be subjected to any simple treatment by linear assumptions. However, mathematical complexity is avoided, in this volume, by approaching the problems from the fundamental physical principles of the device.

The theory of magnetic amplifiers is limited in this text to single phase operation. It begins with the simplest form of a magnetic amplifier, the ordinary saturable reactor. Without changing the viewpoint, higher forms of magnetic amplifiers are evolved in logical order, culminating in the saturable-reactor circuit with blocked intrinsic feedback, known as the amplistat or magamp.

Because magnetic amplifiers are often used in closed-loop systems, they are also treated in servo-block diagram form suitable for immediate integration into a servo system.

The magnetic amplifier, though presenting a simple physical configuration, is highly complex in its internal mechanics. Its operation is dependent on a great number of factors. Contrasting this with the orderly presentation of the material in this volume, one can only conclude that such a commendable work could only be the product of an extremely intimate knowledge of the devices.
THOMAS J. BOWDEN

LECTURES ON PARTIAL DIFFERENTIAL EQUATIONS, by I. G. Petrovsky. Translated from the Russian by A. Shenitzer. 245 pages, 16 × 24 cm. New York, Interscience Publishers, Inc., 1954. Price, $5.75.

Professor I. G. Petrovsky is an outstanding representative of the most impressive active mathematical life of Russia and has made very notable contributions to the field of partial differential equations.

The original, Russian, text of this was published in 1950 and grew out of a series of lectures presented by the author to students of mathematics at the Moscow State University.

The subject matter is subdivided in the conventional manner: introduction; classification of equations; hyperbolic equations; elliptic equations; parabolic equations. Save in the introductory section, only linear equations of the second order in one unknown function which are either hyperbolic, or elliptic, or parabolic in the whole region under consideration are studied in greater detail. The author's principal aim is to expose the reasoning underlying the various methods of solution of equations of these types, rather than to produce a treatise covering the entire field.

The *leitmotif* of the entire exposition is the "Cauchy problem" of partial differential equations and its generalizations. Having set the stage in the introductory section by detailed proofs of the Cauchy-Kowalewski and the Holmgren theorems, followed by a discussion of the reduction of equations to appropriate canonical forms, the author proceeds to the consideration of equations of the above-mentioned three types.

The author discusses in detail the "reasonables" of the Cauchy problem for non-analytic functions in the case of hyperbolic equations and introduces the reader to the concept of "generalized solutions" due to S. L. Soboler. In essence, a generalized solution obeys the differential equation, but satisfies the boundary conditions only "in the mean" in some appropriately defined sense. The reasonableness of this approach is justified by the inherent indeterminacy of the physical boundary conditions, which can be specified only within some given limit of resolution.

The uniqueness of the solution of the Cauchy problem for the wave equation and the explicit form of solutions of this equation are considered next. This is followed by a detailed proof of the theorem that *every non-singular linear transformation of the variables* t, x_1, x_2, x_3, *with real constant coefficients which does not change the form of the wave equation is a combination of a Lorentz transformation, a translation of the origin in the space* (t, x_1, x_2, x_3), *and a similarity transformation in that space.* And by a discussion of the mathematical foundations of the special principle of relativity.

A discussion of the use of Cauchy methods in the case of problems confined to finite bounded regions of space is followed by a detailed exposition of the "Fourier method" of separation of the variables appropriate to these problems leading to the concept of eigenvalue and eigenfunction. A considerable amount of space is devoted to the properties of the latter and to existing methods of their determination.

The elliptic and the parabolic equations are treated in a similarly thorough manner. In the case of the former, the author gives proofs of the existence of unique solutions of the (interior and exterior) Dirichlet and Neuman problems for the Laplace equation, and discusses the properties of the harmonic functions.

Another section is devoted to the exposition of the method of solution of these boundary-value problems by means of integral representations ("potential theory"). This is the only reference to the integral methods in the book, save for a rather brief treatment of the Green function included in the discussion of the Fourier method mentioned above.

A presentation of the method of approximate solution of the Dirichlet problem by the method of finite differences concludes this section.

The material relating to parabolic equations is very brief and is essentially confined to proofs of the uniqueness of solutions of the first and the Cauchy boundary problems in this case.

In all, this reviewer subscribes wholeheartedly with the comment appearing in the translator's note: "The presentation of the material is rigorous and, at the same time transparent and readable. Petrovsky recognizes the value of heuristic and physical considerations, but at the same time maintains high mathematical standards. His comments on and discussion of various aspects of the problems under consideration cannot but deepen the students' understanding and appreciation of these problems."

OTTO R. SPIES

BOOK NOTES

FUNDAMENTAL FORMULAS OF PHYSICS, edited by Donald H. Menzel. 765 pages, 16 × 22 cm. New York, Prentice-Hall, Inc., 1955. Price, $10.00.

The contributors to this compilation of physics formulas are top-flight men in their respective fields. The book covers completely the fundamental formulas of: Basic Mathematics, Statistics, Nomograms, Physical Constants, Classical Mechanics, Special Theory of Relativity, General Theory of Relativity, Hydrodynamics and Aerodynamics, Boundary Value Problems in Mathematical Physics, Heat and Thermodynamics, Statistical Mechanics, Kinetic Theory of Gases, Electromagnetic Theory, Electronics, Sound and Acoustics, Geometrical Optics, Physical Optics, Electron Optics, Atomic Spectra, Molecular Spectra, Quantum Mechanics, Nuclear Theory, Cosmic Rays and High-Energy Phenomena, Particle Accelerators, Solid State, Theory of Magnetism, Physical Chemistry, Astrophysics, Celestial Mechanics, Meteorology and Biophysics.

In such a work it is inevitable that some formulas are duplicated in different fields. Some of the chapters are embellished with explanatory text material while others merely list the pertinent formulas. The collection into one volume of such a formidable amount of data fills a real need among scientists. The book will be used not only as a reference work, but as an aid to a quick review of a subject.

ELECTRONS, ATOMS, METALS AND ALLOYS, by William Hume-Rothery. Revised edition, 387 pages, illustrations, 15 × 22 cm. New York, Philosophical Library; London, Iliffe & Sons, Ltd.; 1955. Price, $10.00.

In the revised edition of this useful book on the electron theory as applied to structure of metals, the text has been corrected to include recent work, although the sections on theory of dislocations and the science of nuclear structure are not completely up-to-date since they lie outside the main scope of the book. The author warns that the equilibrium diagrams should not be used for accurate measurement without checking recent work for modifications.

INTRODUCTORY APPLIED PHYSICS, by Norman C. Harris and Edwin H. Hemmerling. 729 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1955. Price, $6.75.

A practical text, this is intended to present physics as a working tool in industry. Emphasis is on present-day applications, though the chapter headings are not indicative of this emphasis. One of the best features of the book is the variety of photographs illustrating applied physics in industry. The text is up-to-date, well organized and should fulfill its purpose of providing an understanding of technical problems in industry.

AN OUTLINE OF ATOMIC PHYSICS, by Oswald H. Blackwood, Thomas H. Osgood and Arthur E. Ruark. Third edition, 501 pages, illustrations, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1955. Price, $7.50.

Extensive revision has been made in this standard text, in order to bring it up-to-date. Over a third of the material is new and the remainder has been entirely rewritten. Even the diagrams have been redrawn, and many new half-tone illustrations are included. The text covers fundamentals of atomic and nuclear physics without wandering off into allied fields. The revised edition will be welcomed by those who have come to depend on the previous editions.

DESIGN IN BRITISH INDUSTRY, by Michael Farr. 333 pages, plates, 20 × 25 cm. New York, Cambridge University Press, 1955. Price, $11.00.

Dealing with British design and production of all sorts of commodities including

household goods, appliances, jewelry, automobiles, radios and light metalwork, and their merchandising, the book evidences the prodigious amount of research conducted by the author. His thesis is that it is just as important for mass-produced articles to be well designed as it is for the product of the individual craftsman. His book covers retail trade practices, arts and crafts schools, the governmental and private societies and committees concerned with design. He goes beyond presentation of facts, in the Part III (Conclusions) and even has Suggestions for Improvements in Part IV. These suggestions include greater recognition of industrial art through the universities, art schools for training industrial designers, establishment of a museum of modern industrial art, and the formation of a new school for industrial art training. The author's views are provocative and will probably not meet with universal approval, especially among the groups he criticizes.

ELECTROPLATING ENGINEERING HANDBOOK, edited by A. Kenneth Graham. 650 pages, diagrams, illustrations, 18 × 26 cm. New York, Reinhold Publishing Corp., 1955. Price, $10.00.

If there is a problem remotely connected with electroplating, the answer will be found in this complete, up-to-date handbook on the subject. Profusely illustrated with charts and diagrams, and full of tables of useful information, the book is a valuable reference for all those interested in electroplating—engineer, manufacturer, designer, user, purchasing agent or company executive. The material, contributed by 43 experts and edited by A. Kenneth Graham, is divided into two parts: one on General Processing Data (including metal surface preparation, plating processes, testing, waste disposal, industrial hygiene and finishing); and the other on Engineering Fundamentals and Practice (including plant layout, care of equipment, exhaust systems, generators, rinsing and drying practices, etc.).

PUBLICATIONS RECEIVED

ALUMINUM PAINT AND POWDER, by Junius D. Edwards and Robert I. Wray. Third edition, 219 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1955. Price, $4.50.

ELEMENTS OF PHYSICS, by George Shortley and Dudley Williams. Second edition, 898 pages, diagrams, 16 × 24 cm. New York, Prentice-Hall, Inc., 1955. Price, $7.95.

ANALYTIC GEOMETRY, by Ross R. Middlemiss. Second edition, 310 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1955. Price, $3.75.

DAILY RIVER STAGES, VOLUME XLVIII 1952. 180 pages, 21 × 27 cm. Washington, U. S. Department of Commerce, 1955. Price, $1.00 (paper).

PLANTS WITHOUT FLOWERS, by Harold Bastin. 146 pages, plates, 15 × 22 cm. New York, Philosophical Library, 1955. Price, $6.00.

THE BIOLOGY OF THE AMPHIBIA, by G. Kingsley Noble. Unabridged republication of the first edition (1931). 577 pages, diagrams, 14 × 21 cm. New York, Dover Publications, Inc., 1955. Price, $4.95 (cloth).

ADVANCED DYNAMICS OF A SYSTEM OF RIGID BODIES, by Edward John Routh. Unabridged republication of the sixth edition (1905). 484 pages, 14 × 21 cm. New York, Dover Publications, Inc., 1955. Price: $3.95 (cloth); $1.95 (paper).

THE DYNAMICAL THEORY OF GASES, by Sir James Jeans. Unabridged republication of the fourth edition (1925). 444 pages, 15 × 23 cm. New York, Dover Publications, Inc., 1955. Price: $3.95 (cloth); $2.00 (paper).

Transistorized Automobile Radio.—The first completely transistorized automobile radio, which will be special equipment on Chrysler and Imperial lines of cars this fall, has been announced by Philco Corporation, Philadelphia.

Production of the auto radio in which eleven tiny transistors replace all electron tubes will commence as soon as tooling has been completed at the Philco plant in Sandusky, Ohio. It will be the first transistorized radio to be built into any line of new cars.

This first auto radio to use transistors will employ the new type exclusive "Surface-Barrier" transistor developed by Philco. The ruggedness and high degree of sensitivity of the "Surface-Barrier" transistor as compared with other types makes the transistorized auto radio possible.

The Philco transistorized auto radio requires less than one tenth of the battery power used by standard auto radios. Even with the motor turned off, the battery drain is so insignificant as to permit radio operation for more than 100 hours.

Reduction in size will be accomplished not only by the use of transistors, each about the size of a pencil eraser, but also by the elimination of the bulky vibrator, power transformer and rectifier.

The new radio will be equipped with a superior type of "search tuner," developed by Philco, which automatically brings in the nearest strong radio signal wherever you may be driving. Tuning by this method is more precise than can be accomplished by hand, and eliminates the necessity for "dial twirling" in quest of broadcast stations.

Automotive engineers and designers are enthusiastic about the opportunities for new design and performance which are opened up by the Philco transistorized auto radio. The use of transistors makes it possible to build more compact receivers, small enough to be installed practically anywhere in the car.

Laboratory and road tests have established the ability of the transistors developed by Philco to withstand shock and vibration many times in excess of that encountered under the most severe road conditions. In thorough road tests, radio performance has been unaffected by the equivalent of hundreds of thousands of miles of car operation.

Both electrical "hum" and mechanical "noises" are eliminated and the transistorized radio starts to operate immediately without "warm up."

"Super Micadon" Capacitors Now in Production.—Cornell-Dubilier Electric Corporation, of South Plainfield, N. J., has announced the production of new "Super Micadon" capacitors. The company believes this new line represents the greatest improvement in mica capacitor design since William Dubilier invented the first capacitor. Representing an entirely new concept in midget mica capacitor design, C-D Super Micadon encapsulated midget mica capacitors provide five to six times the capacity possible in conventional units of the same size. Other features formerly impossible with conventional capacitors include 10 to 35 times higher insulation resistance,

nearly 20 times the moisture resistance, and up to 19 times greater life expectancy.

The remarkable performance of C-D's "Super-Micadon" capacitors is the result of unusual fabrication processes, up to 40 per cent less inductance, parallel plug-in leads, a unique distortion-free element construction, and a new type of hermetic seal and molded-plastic case combination that exceeds Mil-C-5-A requirements.

Improvements in Solar Battery.—The efficiency of the Bell Solar Battery has been practically doubled in the one year since its announcement, Bell Telephone Laboratories have disclosed. The battery is man's first successful device for converting sunlight directly into substantial amounts of electricity.

Bell scientists, who had already achieved a 6 per cent efficiency with the battery when it was first shown, have since steadily increased its efficiency until today they have experimentally obtained 11 per cent—an efficiency comparable to that of the best gasoline engines and more than 20 times higher than that of the best photovoltaic devices available.

The Laboratories team responsible for the increased efficiency and further development of the Bell Solar Battery includes K. D. Smith, E. J. Stansbury, C. J. Frosch and D. F. Ciccolella.

Trial models of the improved battery will have their first practical test as part of the telephone system this summer in Americus, Georgia, where controlled experiments will determine the usefulness of the battery in supplying power to amplifier stations on rural telephone lines.

The Bell Solar Battery is an extremely simple looking apparatus made of wafers of especially prepared silicon. Silicon, one of the world's most abundant elements, is a semiconducting material that is extremely sensitive to light. The improved "cells" or silicon wafers can now be electrically linked together to deliver power from the sun at a rate of up to 100 watts per square yard of silicon surface.

The increased efficiency of the battery has been achieved primarily by refinements in the fabrication process, aimed at reducing internal losses in the individual cells.

The battery was invented by a three-member team of Bell Laboratories scientists—Gerald L. Pearson, Calvin S. Fuller and Daryl M. Chapin—and announced April 25, 1954.

Although the Bell Solar Battery is a relatively new device and still in the research stage, small solar converters like those now being produced experimentally at the Laboratories hold great promise for future use in communication systems where there are small power requirements. Research also indicates that these batteries can be used as power supplies for low-power portable radio and similar equipment.

Key to the Laboratories' technique for producing the batteries is the controlled introduction of a foreign element, boron, into a microscopic layer near the surface of a thin slice of arsenic doped silicon.

Treatment under gas at high temperatures permits the introduction of minute traces of boron into the atomic structure at the surface of the silicon. Introduced at a precise rate and under carefully controlled conditions, the boron reaches a depth of less than 1/10,000 of an inch.

This thin layer and the remainder of the silicon wafer form what is known as a p-n junction, the heart of the battery. This same type of junction built into a single crystal of germanium is

the basis for the original junction transistor, another invention of the Laboratories.

Color TV System for Medical Use.—The first compatible color television system designed specifically for medical use has been demonstrated by the Radio Corporation of America. Introduced at an exhibition sponsored by the National Academy of Sciences at its annual meeting, the system is built around a compact new color TV camera employing three Vidicon pickup tubes similar to the type used widely in industrial black-and-white television systems.

The compactness of the new camera will permit its use in televising surgical operations of particular interest to the medical profession. In addition, a simple lens fitting converts the camera for use with a light microscope, permitting color transmission of microscopic studies vital in pathology and biology. The compatibility of the system makes it possible for the pictures to be seen on standard home-type color TV receivers with no alteration.

It was pointed out that the new medical color TV system is an outgrowth of research in industrial television. While it has been designed specifically for medical applications, it may be adapted to a variety of industrial or other uses where high light levels are not objectionable.

Besides the novel camera, the system includes three compact units packaged in uniform suitcase style and weighing from 40 to 50 pounds each. These units provide for camera control, monitoring, and a regulated power supply. To feed the signal into ordinary color receivers without alteration or to provide a signal compatible with standards approved by the Federal Communications Commission for broadcast, a fourth unit—a colorplexer of similar size—is required.

Emphasizing the portability of the entire chain, the scientists pointed out that the camera itself is somewhat smaller and lighter than those used for black-and-white commercial television.

Three Vidicon pickup tubes—one each for the red, green and blue primary colors of color TV—are arranged in a vertical array within the camera. A system of dichroic mirrors is provided to divide the light from the televised scene into the three separate colors, and each of the Vidicon tubes is equipped with its own lens. The camera is focused remotely by a small reversible motor that moves the entire Vidicon assembly.

The camera may be mounted in various ways, and its associated control, monitor and power units can be placed some distance away. In surgical application, for example, these three units could be placed outside the operating room.

For use with the light microscope, a special lens is simply inserted in a bracket on the camera, permitting it to pick up the image from a microscope placed on a table directly below. With this arrangement, the camera does not need to be lowered to the eyepiece of the microscope, although such an arrangement may be made for the sake of compactness if the camera is to be used exclusively with a microscope.

Superelay, New Lab Control Device.—A versatile new electronic laboratory control device known as the Superelay, which has been tested more than 28 million times without a breakdown, has been introduced by the Chicago Apparatus Company for use in controlling high current units such as incandescent lamps, heating elements, solenoids, and others.

The Superelay has a large variety of applications in the laboratory. In addition to the usual constant temperature applications, it is suitable for pressure and water-level control, and direct photo-tube operation for control and counting operations. It is claimed to be the only relay which is "fail safe."

The CA-Superelay is capable of switching steady current loads as high as 15 amperes with less than five microamperes passing through the contacts of the sensing device. Contactor or photo-cell input may be employed for either forward or reverse operation.

It features a dual input strip which permits the user to operate the relay with either normally open or normally closed sensing elements and outlets, thus providing many different possible connections and insuring "fail safe" laboratory hookups.

The CA-Superelay also incorporates an additional AC outlet for direct stirrer or other operations, a sensitivity control and a "series" connected outlet. The "series" outlet provides a means of avoiding relay trouble caused by high inrush current of incandescent loads and permits the user to provide a marginal energy output of such an incandescent lamp.

The CA-Superelay is useful for temperature control devices using bimetallic strip and mercury thermoregulators as well as resistance and incandescent loads. The use of a d-c. control circuit in the unit provides for operation with very long input leads and also allows the incorporation of external time delay circuits.

Submersible Motor for Gasoline Pumps.—A new submersible pump motor, designed to operate safely while immersed in high-octane gasoline, has been announced by the General Purpose Component Motor Department of the General Electric Company.

According to company engineers, the new motor was developed to help solve some of the problems attendant upon the growing use of high-octane gasolines, whose extreme volatility makes standard methods of pumping hazardous, expensive, and inefficient at varying altitudes and temperatures.

The result of thousands of hours of engineering research and field testing, the new motor has been slimmed down to fit inside a $3\frac{1}{2}$-in. gasoline pipe. Since it was early discovered that it would be impossible to keep gasoline from seeping into the motor, it is designed to be flooded with gasoline, the fluid acting as a coolant and bearing lubricant. The windings and stator core are protected from ill effects of the gasoline by being completely sealed in an impervious epoxy resin material.

For added motor protection and fire safety, a thermostat is provided which disconnects the motor from the line shortly after the bulk tank is pumped dry.

The motor is rated $\frac{3}{4}$ hp., 230 volts, 60 cycles, 3450 rpm. It will absorb a 50-lb. thrust, according to G-E engineers, when the pump thrust is away from the motor. Inspection and approval has been obtained from the Underwriters' Laboratories.

Titanium Purification by Zone Refining.—Scientists at the Westinghouse Research Laboratories are purifying titanium and other hard-to-get metals by imprisoning the molten metal inside a cage of its own making. The process, called cage zone refining, uses a unique method to melt a bar of metal while the metal acts as its own crucible, thus preventing contamination by any containing vessel. Object of the process is to prepare super-pure metals.

In zone refining which has become a standard process for purifying certain metals, a bar is melted progres-

sively from one end to the other. The new technique applies this valuable refining process to metals which are so active at high temperatures that they react with any crucible in which they can be heated.

The success of zone refining depends upon the fact that most impurities in metals have a preference for either the liquid or solid state of the metal. Iron, a common impurity in titanium, has a preference for the liquid state of titanium. When a bar of impure titanium is melted progressively from end to end, the iron concentrates in the liquid titanium and follows the molten zone to the end of the bar. This end is then cut off and discarded, leaving the rest of the titanium bar more pure. Each time the process is repeated, more impurities are removed.

Titanium, at its high melting point of over 3000 F., is one of the most active metals known. It not only oxides rapidly but it reacts chemically with metal, graphite, or ceramic containers. The new technique eliminates such sources of contamination during refining.

The new Westinghouse process does this in a unique way. In a low-pressure, inert atmosphere of argon or helium gas, a square bar of impure titanium stands upright on a metal platform. The platform is slowly raised, lifting the bar through the heating coil.

A high-frequency current at 10,000 cycles per second flows in the coil and induces large electric currents inside the bar. These currents, called eddy currents, flow against the electrical resistance of the metal, causing the bar to melt from the inside. The four corners of the bar are less affected by the induced currents and rapidly lose what heat they possess to the surrounding atmosphere. Thus the corners remain rigid and act as a cage within which the molten titanium is "imprisoned."

In one variation of the new Westinghouse process, a round bar is machined so that several ridges or fins run lengthwise along its surface, much like those on the fluted columns of certain buildings. These unmelted ridges then form the cage, and since they can be placed at will, they permit longer bars of larger diameter to be purified. The Westinghouse scientists foresee the technique being applied to bars as much as three inches in diameter and of any desired length.

Gold Bullets Fight Cancer.—A London firm is now making for export "gold guns" for use in the treatment of cancer. The gun, developed by the staff of the Royal Hospital, Chelsea, London, is shaped like a heavy automatic. It fires radio-active gold isotopes into affected areas of the body, after being set to eject the bullets to the depth of the tumour which can be seeded with any number of charges. As the gold isotopes lose their radioactivity after ten days or so, they can be left in the body without harm to the patient. The gun's long barrel takes 15 gold isotopes bullets each a tenth of an inch long. The development of the gun was made possible by considerable new advances in isotope technique at Harwell, Britain's Atomic Energy Establishment. The gold bullets are "cooked" in atomic piles at Harwell and sent to hospitals in special magazines and loading trays for safe handling.

High Speed Laboratory Mill.—A high-speed laboratory mill, with a wide range of uses, but particularly suited to the paint industry and the paint manufacturers' laboratory, has just been put into production in England. The manufacturer, Steele and Cowli-

shaw, Ltd., states the new mill will grind and disperse materials in one-tenth of the time normally taken by the orthodox ball or pebble mill. It will also mill to semi-paste consistency, grind materials wet or dry, and can, if necessary, process four different substances simultaneously. Formulations processed in this new mill can be reproduced in bulk production with identical characteristics; the only variable is the time factor. Applied to the paint industry, it will, for example, grind and disperse a concentrated charge of color pigment and medium in approximately four hours, a process which normally takes at least 40 hours in an orthodox ball or pebble mill. In five, six or seven hours' milling time, it will produce a much superior dispersion.

Butyro Refractometer.—A new Butyro Refractometer designed to increase the speed and accuracy of purity tests made on edible oils has been introduced by the Bausch & Lomb Optical Co., Rochester, N. Y.

The instrument will find many applications in plants processing edible oils, such as cooking oils, soybean oils, vegetable oils, shortenings, butter, and oleo, the manufacturer said.

To determine butyrol number, oil samples are placed between two prisms mounted in a fixed horizontal position and light is refracted through the liquid oil. Direct readings of the refraction are made through the micrometer-equipped eyepiece in butyrol number accurate to 0.1 per cent. The horizontal sample mounting position permits an unobstructed view of the oil sample and allows easy loading and cleaning of the instrument.

The new B&L Butyro Refractometer has many features not available with previous instruments. Temperatures are kept constant for accurate readings with spiral flow copper heat exchangers around the prism boxes. Possibility of contaminating samples is very remote because the instrument's clean design eliminates slits, corners or holes. The prisms of the new refractometer are mounted with special cement in corrosion-proof housings for long life. Spare prisms are also available.

Technicians using the new instrument will be able to make many more butyrol determinations because both line and fixed scale are visible simultaneously in the eyepiece and samples are quickly changed. The built-on light source also assures simplicity of operation. Samples can be read by either transmitted or reflected light.

Lifesaving Fuel Tank.—Rubberized nylon has been made into an airplane fuel tank designed to resist rupture at high crash speeds. The tank successfully passed a recent test during which it was filled with water, encased in an airplane wing, and catapulted 300 feet into a steel-backed sandbag barrier. Since many crash victims survive the impact only to die in flames when fuel tanks explode, the new rupture-resistant tank is expected to save many lives. Oil products provide safety engineers with many of their most useful materials.

New Role for "Dacron."—"Dacron" polyester fiber is being woven into a heavy-duty jacket for a fire hose developed especially for the oil and chemical industries. The natural fibers formerly used were no match for the oils, acids, and extreme temperatures found around oil and chemical plants; hose deteriorated quickly. "Dacron" shows excellent resistance under these conditions and also resists mildew from dampness. The "Dacron" jacket, impregnated with neoprene cement, covers a neoprene tube. The combination of these two oil products results in a

hose of little more than half the weight of the conventional type, yet it withstands more pressure and higher temperatures. Its lightness and flexibility speed fire-fighting, making possible untold savings of property, and sometimes of lives.

Scissors-Type Rafter Design.—A new scissors-type Teco trussed rafter design, having a lower roof slope of 8-in-12, instead of the 12-in-12 slope in previous designs, has been developed for architects and other designers by Timber Engineering Company, research affiliate of the National Lumber Manufacturers Association, and longtime authorities on the use of engineered timber in light and heavy construction.

The new typical design is particularly adaptable to small churches, chapels, and similar buildings of contemporary design. It enables the structures to retain their more or less traditional high roof pitch and steep ceiling slope, yet enables both to be lowered for more conformance with the current trend toward lower roofs.

Architects will find the new design equally adaptable to commercial and industrial buildings requiring clear span construction, a pitched roof, and greater floor-to-ceiling clearance. It can also provide sloped ceilings required by architectural interior styling of some present-day dwellings.

The new typical design provides data for spans from 20 feet through 36 feet, and spacings of two feet on center. All data are based on the use of Teco wedge-fit timber connectors in conforming grooves. In addition to specifications and a list of required materials for the scissors trussed rafter, the design includes anchorage detail showing how to tie the roof down more securely with Trip-L-Grip framing anchors.

Earmuffs to Protect Hearing.—A revolutionary type of earmuff that makes men look like something from a dream of outerspace has been developed in Worcester. The device fits over the head like a phone set and covers each ear to muffle sound of every frequency.

It was developed at Worcester Polytechnic Institute by Prof. William D. Wadsworth, several graduate students, and the David M. Clark Co. The latter financed the project which had its start two years ago.

One of its proven uses is in jet engine testing where sound alone can vibrate the leather soles on a man's shoes.

Testing in a soundproof room at WPI determined that the best design was an elliptical shape for the earmuff. The outer surface is of reinforced fiber glass. A plastic foam ring seals the muff over the ear, and a plastic sponge material with a backing of chamois lines the inside.

The device is more effective than earplugs, which have low acceptability among workers because of discomfort, according to Professor Wadsworth.

With the new muffs, the only sound transmitted is through the bone structure of the head, Forrest R. Poole of the Clark Company says. Experiments now are underway to eliminate sound from penetrating this bone structure. It will probably be necessary to cover the entire head, Professor Wadsworth believes.

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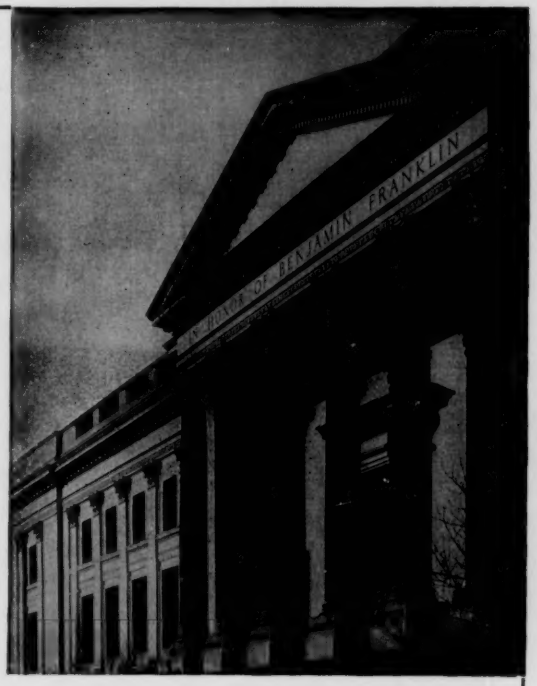

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